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EQUITY HOME BIAS AND NOMINAL RIGIDITY

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Abstract

This paper examines optimal portfolio decisions in a monetary open-economy DSGE model. In a complete market environment, Engel and Matsumoto (2005) find that sticky price can generate equity home bias. However, their result is sensitive to the structure of the financial market. In an incomplete market environment, we find "super home bias" in the equilibrium equity portfolio, which casts doubt on the ability of sticky price alone in describing the observed equity portfolios. We further show that introducing sticky wage helps to match the data.

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1 Introduction

Typical investors hold too little of their wealth in foreign assets relative to the predictions of standard financial and macroeconomic theory. According to French and Poterba (1991) and Tesar and Werner (1995), the percentages of aggregate stock-market wealth invested in domestic equities at the beginning of the 1990s are well above 90% for the US and Japan, and are around 80% in the UK and Germany. Based on the portfolio data from Kraay et al. (2005), Kollmann (2005) shows that the average locally owned capital share for 17 OECD countries is 91% in 1997. More recently, Heathcote and Perri (2007) report that foreign assets account for only around 25% of the total value of the assets owned by the U.S. residents over the period 1990-2004. This widespread lack of diversification across countries, named equity home bias, has become a major empirical puzzle in international finance.

In a complete financial market environment, Engel and Matsumoto (2005) find that the presence of nominal price rigidity can help explain the equity home bias puzzle, because it generates a negative correlation between the labor income and the profit of domestic firms with respect to the productivity shocks. Within a similar open economy framework, this paper examines the optimal portfolio decisions focusing on the impact of two additional frictions: first, incomplete financial market; second, nominal wage rigidity.

We find that in an incomplete market environment, the labor income and the profit of domestic firms are negatively correlated in response to not only the productivity shocks but also the monetary shocks. Therefore, the equilibrium portfolios require an aggressive investment position in domestic equities. This “super home bias” result weakens the ability of sticky price alone in describing the observed equity holdings. Introducing sticky wage can help match the data because the labor income and the profit of domestic firms become positively correlated with respect to monetary shocks in this case. Hence, it is optimal for households to hold some positive amount of foreign equities. With incomplete financial markets and nominal rigidities in both the goods prices and the wage rates, this model predicts home bias for a wide range of parameterization which are often used in the macroeconomics literature.

Note that it is the conditional correlation between returns to human capital and returns to domestic equities that determines the equilibrium portfolios. The unconditional correlation can vary substantially without changing the underlying portfolio structures qualitatively. This sheds light on the mixed empirical evidence on the correlation between returns to human capital and returns to domestic equities. Using annual OECD data, from Japan, Germany, the United Kingdom, and the United States, Baxter and Jermann (1997) find that human capital returns are highly correlated with domestic capital returns between 1960 and 1993. However, using slightly different measures based on annual OECD data between 1970 to 1992, Bottazzi et al. (1996) find that human capital returns are negatively correlated with domestic capital returns for most OECD countries except the United States. Obviously, it is not clear whether home bias is puzzling or not by just looking at the unconditional correlation. What is important is the conditional correlation. Gali (1999) and Gali and Rabanal (2004) find that the conditional correlation between labor hours and productivity is negative in response to technological shocks, while the unconditional correlation is positive. These results are further confirmed by Francis and Ramey (2005a,b) and Rotemberg (2003).

There is a large literature that seeks resolutions to the equity home bias puzzle.[1] Potential explanations range from barriers to international capital movements to frictions that justify the observed portfolios as optimal risk management decisions. This paper builds on one thread of the study that emphasizes the importance of hedging against non-traded labor income risk.

Baxter and Jermann (1997) show that returns to human capital are positively correlated with returns to domestic equities but not with returns to foreign equities. As the labor

[1] See Lewis (1999) for a comprehensive survey on this literature.

income risk is non-diversifiable and the labor income accounts for more than half of the total income, investors should take large short positions in domestic assets. The implied equity portfolios are more foreign-weighted than what a classical endowment economy, such as Lucas (1982), would predict. Indeed, as long as non-traded labor income is more correlated with the domestic stock market than with the foreign stock market, the puzzle becomes even worse. In the flexible price case of our model, we corroborate the above results and show that they are identical in the sense that both predict a perfect pooling (each country receives half of the world output).

Jermann (2002) characterizes optimal portfolios in a multi-country general equilibrium model with endogenous labor - leisure choice and with non-separable preference between consumption and leisure. The return to human capital and the return to domestic equity are still positively correlated. However, with consumption and leisure being substitutes, consumption is highly valued in periods when work effort is high. Therefore, a domestic claim provides the right hedge. Heathcote and Perri (2007) studies a two-goods general equilibrium model with investment. In their framework, each country specializes in production of a final good that uses both local and imported intermediate inputs. Following a positive domestic productivity shock, home output and demand for labor increase. Home investment goes up as well, which reduces the dividend paid on home equities. Thus, the labor income is negatively correlated with the return to domestic equities. Home bias arises because domestic stocks make a good hedge against non-diversifiable labor income risk.

This paper also falls under a growing literature on the analysis of country portfolio compositions in dynamic general equilibrium models. Kollmann (2005) and Hnatkovska (2005) employ non-monetary dynamic general equilibrium framework and focus on the equity home bias puzzle. Kollmann (2005) generates portfolio home bias in an endowment economy with home bias in consumption and complete markets. Hnatkovska (2005) shows that equity home bias can arise naturally in the presence of non-traded consumption risk, consumption home bias, and incomplete asset markets. To solve for the endogenous portfolio composition, Evan and Hnatkovska (2005) proposes a numerical approximation method, while Devereux and Sutherland (2006) develops a second-order approximation method. Both approaches are designed to handle dynamic general equilibrium models, especially those with incomplete financial markets. This paper applies the methodology of Devereux and Sutherland (2006).

The paper is organized as follows. Section 2 presents the model. Section 3 describes the solution to the model and compares the equilibria under different market configurations. Section 4 analyzes the effect of sticky wage. Sections 5 concludes.

2 The Model

The world is assumed to exist for a single period and to consist of two countries, which will be referred to as the home country and the foreign country. Each country is populated by agents who consume a basket of home and foreign produced goods. Each agent, using a linear technology in labor, is a monopoly producer of a particular differentiated product. The world population is normalized to have a measure of one. Home agents are indexed by $h \in [0, \frac{1}{2}]$ and foreign agents are indexed by $f \in [\frac{1}{2}, 1]$.

A fraction κ of agents in each country set prices before the realization of shocks. They are contracted to meet demand at the pre-fixed prices. Other agents in the economy can set prices after shocks are realized. All prices are assumed to be set in the currency of producers. Thus, there is full exchange rate pass-through to prices paid by consumers. Agents supply homogeneous labor.[2] Prior to the realization of shocks, they can trade in a range of financial assets. The financial market structure and the payoff to each asset are defined in Section 2.3. Each country faces two types of shocks: the productivity and the money supply shocks.[3]

The detailed structure of the home country is described below. The foreign country has

[2]We first look at the effect of nominal price rigidity. Nominal wage rigidity will be introduced later.
[3]Results are identical if we assume money demand shocks.

an identical structure. Where appropriate, foreign variables are denoted with an asterisk.

2.1 Consumers

All agents in the home country have utility functions of the same form

$$U = \frac{C^{1-\rho}}{1-\rho} + \chi log \frac{M}{P} - \eta \frac{L^{1+\psi}}{1+\psi} \tag{1}$$

where C is the consumption index, defined across all home and foreign goods, M denotes the end-of-period nominal money holding, P is the consumer price index (CPI), L is the labor supply, ρ ($\rho \geq 0$) is the coefficient of relative risk aversion, χ is the coefficient of real balance, η is the coefficient of labor supply, and ψ ($\psi \geq 0$) is the elasticity of labor supply.

There are two stages to the household decision problem. Before shocks are realized, households choose portfolio positions out of available assets to maximize expected utility, $E\left[U(C, \frac{M}{P}, L)\right]$, subject to

$$\sum_{k=1}^{n} \alpha_k = 0 \tag{2}$$

where α_k represents the real holding of asset k, and n is the total number of assets. All real variables in this paper are defined in terms of home consumption basket. Each country starts with zero net wealth.

After shocks are realized, households choose consumption, labor supply, and money balances, in order to maximize ex-post utility, $U(C, \frac{M}{P}, L)$, subject to

$$M + PC = M_0 + P_H Y_H + P \sum_{k=1}^{n} \alpha_k r_k + T \tag{3}$$

where M_0 is the initial nominal money holding, P_H is the aggregate price of home produced goods, r_k is the real aggregate rate of return on asset k, and T is a lump-sum government transfer. Y_H is the world demand for aggregate home produced goods

$$Y_H = \frac{1}{2}\left(\frac{P_H}{P}\right)^{-\theta}(C + C^*) \tag{4}$$

where θ is the elasticity of substitution between home and foreign goods.

Firms' revenues are used to pay wages and profits

$$P_H Y_H = wL + \Pi \tag{5}$$

where w and Π denote the wage and the profit (dividend), respectively.

Notice that home agents first receive all profits from domestic firms. If an international equity market exists, claims to home profits may then be transferred to foreign consumers via trade in equity shares.[4]

Moreover, we define Y as the home aggregate real production income

$$PY = P_H Y_H \tag{6}$$

Combined with the government budget constraint

$$M - M_0 = T \tag{7}$$

we can rewrite home household's budget constraint (3) in real terms

$$C = Y + \sum_{k=1}^{n} \alpha_k r_k \tag{8}$$

M is an i.i.d. stochastic money supply shock with $E(logM) = 0$, $Var(logM) = \sigma_M^2$, and

[4]Alternatively, we can assume that profits proceed directly to shareholders. There is no fundamental difference between the two modeling approaches.

$logM \in [-\epsilon, \epsilon]$.

The consumption index C for home agents is defined as

$$C = \left(\frac{1}{2}\right)^{\frac{1}{\theta-1}} \left(C_H^{\frac{\theta-1}{\theta}} + C_F^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}} \tag{9}$$

C_H and C_F are indices of home and foreign produced goods

$$C_H = \left[\left(\frac{1}{2}\right)^{\frac{1}{\phi}} \int_0^{\frac{1}{2}} C_H(h)^{\frac{\phi-1}{\phi}} dh\right]^{\frac{\phi}{\phi-1}}, \quad C_F = \left[\left(\frac{1}{2}\right)^{\frac{1}{\phi}} \int_{\frac{1}{2}}^{1} C_F(f)^{\frac{\phi-1}{\phi}} df\right]^{\frac{\phi}{\phi-1}} \tag{10}$$

where ϕ ($\phi > 1$) is the elasticity of substitution between individual home (or foreign) goods.

The aggregate consumer price index for home households is

$$P = \left(\frac{1}{2}\right)^{\frac{1}{1-\theta}} \left(P_H^{1-\theta} + P_F^{1-\theta}\right)^{\frac{1}{1-\theta}} \tag{11}$$

where P_H and P_F are the price indices for home and foreign produced goods

$$P_H = \left[2 \int_0^{\frac{1}{2}} P_H(h)^{1-\phi} dh\right]^{\frac{1}{1-\phi}}, \quad P_F = \left[2 \int_{\frac{1}{2}}^{1} P_F(f)^{1-\phi} df\right]^{\frac{1}{1-\phi}} \tag{12}$$

The law of one price implies that $P_H(h) = P_H^*(h)S$ and $P_F(f) = P_F^*(f)S$ for all h and f. An asterisk indicates that the price is in foreign currency. S is the nominal exchange rate defined as the domestic price of foreign currency. Because there is no home bias in consumption, purchasing power parity (PPP) holds, *i.e.* $P = SP^*$.

Given prices and the total consumption C, home consumers' optimal demands for home and foreign goods are

$$C_H = \frac{1}{2}\left(\frac{P_H}{P}\right)^{-\theta} C, \quad C_F = \frac{1}{2}\left(\frac{P_F}{P}\right)^{-\theta} C \tag{13}$$

$$C_H(h) = 2\left[\frac{P_H(h)}{P_H}\right]^{-\phi} C_H, \quad C_F(f) = 2\left[\frac{P_F(f)}{P_F}\right]^{-\phi} C_F \tag{14}$$

The remaining first order conditions are

$$\begin{aligned} E\left(r_1 C^{-\rho}\right) &= E\left(r_n C^{-\rho}\right) \\ E\left(r_2 C^{-\rho}\right) &= E\left(r_n C^{-\rho}\right) \\ &\vdots \\ E\left(r_{n-1} C^{-\rho}\right) &= E\left(r_n C^{-\rho}\right) \end{aligned} \tag{15}$$

$$\eta L^{\psi} = \frac{wC^{-\rho}}{P} \tag{16}$$

$$\frac{\chi}{M} = \frac{C^{-\rho}}{P} \tag{17}$$

Equation (15) is the set of arbitrage conditions, which indicates that portfolio choices are optimal only when the expected returns on all assets are equalized in terms of utility. Equations (16) and (17) are the standard intra-temporal labor-leisure choice function and the money demand function.

2.2 Firms

Firms engage in monopolistic competition. Each firm in the home country produces a specific type of goods indexed by h with a linear technology in labor $Y_H(h) = AL(h)$. A is an i.i.d. stochastic technological shock with $E(logA) = 0$, $Var(logA) = \sigma_A^2$, and $logA \in [-\epsilon, \epsilon]$. By assumption, a fraction κ of firms have to set prices in advance and the rest can set prices after shocks are realized.

The profit maximization problem of the home firm i with pre-set price is

$$Max \;\; E\{D(i)\left[P_{pre,H}(i) - \frac{w}{A}\right]\left[Y_{pre,H}(i) + Y^*_{pre,H}(i)\right]\}$$

where $D(i)$ is the stochastic discount factor for firm i, $Y_{pre,H}(i)$ and $Y^*_{pre,H}(i)$ are the demand for good i from the home and foreign markets

$$Y_{pre,H}(i) = \left[\frac{P_{pre,H}(i)}{P_H}\right]^{-\phi}\left(\frac{P_H}{P}\right)^{-\theta} C, \quad Y^*_{pre,H}(i) = \left[\frac{P^*_{pre,H}(i)}{P^*_H}\right]^{-\phi}\left(\frac{P^*_H}{P^*}\right)^{-\theta} C^* \quad (18)$$

Because firms of each type are all alike, they will set identical prices in equilibrium. Hence, the optimal pre-set price of home goods is

$$P_{pre,H} = \frac{\phi}{\phi-1}\frac{E\left[D\frac{w}{A}X_H\right]}{E\left[DX_H\right]} \quad (19)$$

where X_H represents the demand for home produced goods

$$X_H = P_H^{\phi}\left(\frac{P_H}{P}\right)^{-\theta}(C+C^*)$$

Notice that we have applied the law of one price and PPP in CPI.

The profit maximization problem of the home firm j with flexible price is

$$Max \quad D(j)\left[P_{flx,H}(j) - \frac{w}{A}\right]\left[Y_{flx,H}(j) + Y^*_{flx,H}(j)\right]$$

where $D(j)$ is the stochastic discount factor for firm j, $Y_{flx,H}(j)$ and $Y^*_{flx,H}(j)$ are the demand for good j from the home and foreign markets

$$Y_{flx,H}(j) = \left[\frac{P_{flx,H}(j)}{P_H}\right]^{-\phi}\left(\frac{P_H}{P}\right)^{-\theta} C, \quad Y^*_{flx,H}(j) = \left[\frac{P^*_{flx,H}(j)}{P^*_H}\right]^{-\phi}\left(\frac{P^*_H}{P^*}\right)^{-\theta} C^* \quad (20)$$

The optimal flexible price of home goods is

$$P_{flx,H} = \frac{\phi}{\phi-1}\frac{w}{A} \quad (21)$$

The price index for home produced goods can be rewritten as

$$P_H = \left[\kappa P_{pre,H}^{1-\phi} + (1-\kappa) P_{flx,H}^{1-\phi}\right]^{\frac{1}{1-\phi}} \tag{22}$$

2.3 Financial Sector

This paper examines two different asset market configurations: (1) Equity Economy (FE), in which home and foreign equities can be traded internationally; (2) Bond and Equity Economy (FBE), in which home and foreign nominal bonds are allowed for trade in addition to their equities. The degree of consumption risk sharing in each financial market depends on the nature of the rest of the model.

Nominal bonds represent a claim on a unit of currency. Equities represent a claim on aggregate profits. The real aggregate rate of return on each asset is defined as following

$$r_B = \frac{1}{q_B P}, \quad r_{B^*} = \frac{Q}{q_{B^*} P^*} \tag{23}$$

$$r_E = \frac{\Pi}{q_E P}, \quad r_{E^*} = \frac{Q\Pi^*}{q_{E^*} P^*} \tag{24}$$

where q_k is the real price of asset k and Q is the real exchange rate, *i.e.* $Q = \frac{SP^*}{P}$.

2.4 Market Clearing

The goods market clearing condition is

$$AL = \frac{1}{2}\left(\frac{P_H}{P}\right)^{-\theta}(C + C^*) \tag{25}$$

The asset market clearing condition is

$$\alpha_k = -\alpha_k^*, \quad k = 1, 2, \cdots, n \tag{26}$$

2.5 Equilibrium

The *equilibrium* comprises a set of prices, P, P^*, P_H, P_F^*, $P_{pre,H}$, $P_{pre,F}^*$, $P_{flx,H}$, $P_{flx,F}^*$, w, w^*, r_k, S, and a set of quantities C, C^*, L, L^*, Π, Π^*, Y_H, Y_F^*, Y, Y^*, α_k, α_k^*, M, M^*, which solves a system of equations (4)-(6), (11), (15)-(17), (19), (21)-(22), (25), and their foreign counterparts, as well as (8), (23)-(24), and (26), given the productivity and the money supply shocks, A, A^*, M, and M^*.[5]

3 Portfolio Decisions

3.1 Solution Method

Using the method developed by Devereux and Sutherland (2006), the equilibrium portfolio choices are solved to a second-order accuracy. The second-order approximation of home portfolio selection equations (15) around the non-stochastic steady state are given by[6]

$$\begin{aligned}
E[(\hat{r}_1 - \hat{r}_n) + \frac{1}{2}(\hat{r}_1^2 - \hat{r}_n^2) - \rho\hat{C}(\hat{r}_1 - \hat{r}_n)] &= 0 + O(\epsilon^3) \\
E[(\hat{r}_2 - \hat{r}_n) + \frac{1}{2}(\hat{r}_2^2 - \hat{r}_n^2) - \rho\hat{C}(\hat{r}_2 - \hat{r}_n)] &= 0 + O(\epsilon^3) \\
&\vdots \\
E[(\hat{r}_{n-1} - \hat{r}_n) + \frac{1}{2}(\hat{r}_{n-1}^2 - \hat{r}_n^2) - \rho\hat{C}(\hat{r}_{n-1} - \hat{r}_n)] &= 0 + O(\epsilon^3)
\end{aligned} \tag{27}$$

Because shocks are symmetrically distributed in the interval $[-\epsilon, \epsilon]$, $O(\epsilon^n)$ represents residuals of an equation approximated to order $n-1$.

[5] By Walras' Law, there is one equation redundant in the system. WOLG, we drop the foreign agent's budget constraint.

[6] Hereafter, $\hat{x} = log(X) - log(\bar{X})$.

Foreign agents face a set of portfolio selection equations similar to (15)

$$
\begin{aligned}
E\left(\frac{r_1}{QC^{*\rho}}\right) &= E\left(\frac{r_n}{QC^{*\rho}}\right) \\
E\left(\frac{r_2}{QC^{*\rho}}\right) &= E\left(\frac{r_n}{QC^{*\rho}}\right) \\
&\vdots \\
E\left(\frac{r_{n-1}}{QC^{*\rho}}\right) &= E\left(\frac{r_n}{QC^{*\rho}}\right)
\end{aligned} \tag{28}
$$

whose second-order approximation is given by

$$
\begin{aligned}
E[(\hat{r}_1 - \hat{r}_n) + \frac{1}{2}(\hat{r}_1^2 - \hat{r}_n^2) - \rho\hat{C}^*(\hat{r}_1 - \hat{r}_n)] &= 0 + O(\epsilon^3) \\
E[(\hat{r}_2 - \hat{r}_n) + \frac{1}{2}(\hat{r}_2^2 - \hat{r}_n^2) - \rho\hat{C}^*(\hat{r}_2 - \hat{r}_n)] &= 0 + O(\epsilon^3) \\
&\vdots \\
E[(\hat{r}_{n-1} - \hat{r}_n) + \frac{1}{2}(\hat{r}_{n-1}^2 - \hat{r}_n^2) - \rho\hat{C}^*(\hat{r}_{n-1} - \hat{r}_n)] &= 0 + O(\epsilon^3)
\end{aligned} \tag{29}
$$

Note that we have applied PPP. By subtracting equations (27) from (29), the set of equations to solve for equilibrium portfolios is obtained

$$
E[\rho(\hat{C} - \hat{C}^*)\hat{r}_x] = 0 + O(\epsilon^3) \tag{30}
$$

It is written in a vector form with $\hat{r}_x' = [\hat{r}_1 - \hat{r}_n,\ \hat{r}_2 - \hat{r}_n,\ \cdots,\ \hat{r}_{n-1} - \hat{r}_n]$.

Condition (30) contains only the second moments of endogenous variables, indicating that solving the portfolio choice to a second-order accuracy only requires the first-order solution of the non-portfolio part of the model. This is because second-order accurate second moments can be computed from first-order solutions for realized variables. Finally, firms' discount factors do not appear in equation (30) or any other equilibrium condition up to the first-order. Therefore, they do not affect the solution of optimal portfolio choices.[7]

3.2 Risk Sharing

Use the log-linearized home CPI,

$$\hat{P} = \frac{1}{2}(\hat{P}_H + \hat{P}_F^* + \hat{S}) + O(\epsilon^2) \tag{31}$$

we can express home real GDP as

$$\begin{aligned} \hat{Y} &= \frac{1-\theta}{2}(\hat{P}_H - \hat{P}_F^* - \hat{S}) + \frac{1}{2}(\hat{C} + \hat{C}^*) + O(\epsilon^2) \\ &= \frac{1-\theta}{2}\hat{\tau} + \frac{1}{2}(\hat{C} + \hat{C}^*) + O(\epsilon^2) \end{aligned} \tag{32}$$

where τ is the home country's terms of trade — defined as the price of exports relative to the price of imports. Because there is no aggregate uncertainty at the world level, country-specific income risks all come from the terms of trade fluctuations.

Moreover, the log-linearized home budget constraint is

$$\hat{C} = \hat{Y} + \tilde{\alpha}'\hat{r}_x + O(\epsilon^2) \tag{33}$$

where $\tilde{\alpha} = \frac{\bar{\alpha}}{\bar{Y}}$. Hence, the consumption difference between home and foreign countries can be written as

$$\frac{1}{2}(\hat{C} - \hat{C}^*) = \frac{1-\theta}{2}\hat{\tau} + \tilde{\alpha}'\hat{r}_x + O(\epsilon^2) \tag{34}$$

Clearly, households' consumption risks originate from uncertainties in their real income, which are further caused by variations in the terms of trade. Suppose home country's terms

[7]The complete log-linearization of the model is presented in Appendix A.1.

of trade deteriorate in response to certain fundamental shocks, $\hat{\tau} < 0$. Home goods become cheaper relative to foreign goods. Given that home and foreign goods are substitutes, the world demand shifts towards home goods. Home output increases, $\hat{Y}_H > 0$, which is called the expenditure switching effect. However, home goods are sold at a relatively lower price and foreign goods cost relatively more. Whether home GDP rises or falls in real terms depends on the strength of this substitution across countries. If the expenditure switching effect is strong enough ($\theta > 1$), home real GDP increases. The exact opposite takes place in the foreign country. As a result, the portfolio decision is essentially about how to optimally hedge against the terms of trade fluctuations. Agents can diversify away at least part of the consumption risks by trading assets across borders — as long as the returns on these assets are somehow correlated with the terms of trade.

Now it is possible to state the optimal portfolio choices under each financial market configuration. The model is symmetric, which implies that in the FB economy, agents in both countries will have bond holdings that sum to zero, and in the FBE economy, their equity holdings and bond holdings will separately sum to zero. Thus, for the home country, we have $\tilde{\alpha}_{FE,E^*} + \tilde{\alpha}_{FE,E} = 0$ in the FE economy, and $\tilde{\alpha}_{FE,B^*} + \tilde{\alpha}_{FE,B} = 0$, $\tilde{\alpha}_{FE,E^*} + \tilde{\alpha}_{FE,E} = 0$ in the FBE economy.

As the aim is to understand the effect of nominal price rigidity on the equilibrium equity holdings, we will focus on the two extremes, in which goods prices are either completely flexible ($\kappa = 0$) or completely sticky ($\kappa = 1$). Any intermediate case with $0 < \kappa < 1$ can be easily inferred from these two scenarios.

3.3 Flexible Price Case: $\kappa = 0$

Table 1 describes the optimal portfolio holdings in the FE and FBE economies when goods prices are fully flexible. The equilibrium asset holdings are identical between the two economies. In this case, the excess return of foreign equities relative to home equities is

$\hat{r}_{E^*} - \hat{r}_E = (\theta - 1)\hat{\tau} + O(\epsilon^2)$.[8] As long as equities can be traded internationally, households achieve perfect consumption risk sharing.

Table 1: Optimal Portfolio Holdings with Flexible Prices

FE	$\tilde{\alpha}_{FE,E^*} = \frac{1}{2}$
FBE	$\tilde{\alpha}_{FBE,B^*} = 0$
	$\tilde{\alpha}_{FBE,E^*} = \frac{1}{2}$

Here, money is neutral. Money supply shocks have no real effect on the economy, as changes in the prices and the exchange rate cancel out completely. The terms of trade respond only to the productivity shocks. There is no need to trade nominal bonds. With the optimal equity positions, households essentially receive half of each country's output, which corresponds to the "full diversification" prediction by Lucas (1982).

The share of domestically owned home equities is given by $\delta_E = 1 - \frac{\bar{\alpha}_{E^*}}{q_E} = 1 - \frac{\tilde{\alpha}_{E^*}}{\Pi/PY} = 1 - \frac{\tilde{\alpha}_{E^*}}{1-\zeta}$, where ζ is the share of labor income in total output at the non-stochastic steady state. When prices are fully flexible, $\delta_E = \frac{1-2\zeta}{2(1-\zeta)}$. As labor income generally accounts for more than half of the total income, $\delta_E < 0$. In other words, home households should not only diversify their portfolios and hold foreign equities, but indeed take a short position in their domestic equities. This reproduces the well-known Baxter and Jermann (1997) result that households should aggressively invest in foreign equities to hedge their non-tradable labor income from the productivity shocks, because the labor income and the profit of domestic firms are highly correlated.

3.4 Sticky Price Case: $\kappa = 1$

Table 2 describes the optimal portfolio holdings in the FE and FBE economies when goods prices are all pre-set. The two market structures accomplish different degrees of international

[8] Detailed derivation see Appendix A.2.

Table 2: Optimal Portfolio Holdings with Sticky Prices

FE	$\tilde{\alpha}_{FE,E^*} = \frac{1}{2}\frac{(\theta-1)(\zeta-1)\{[\theta(1+\psi)+\rho(\theta-1)]\zeta+(1-\theta)\}\sigma_M^2}{\zeta^2(\psi+1)^2[1+\rho(\theta-1)]\sigma_A^2+[\theta\zeta(\psi+1)+(1-\theta)]\{[\theta(\psi+1)+\rho(\theta-1)]\zeta+(1-\theta)\}\sigma_M^2}$
FBE	$\tilde{\alpha}_{FBE,B^*} = \frac{1-\theta}{2}$
	$\tilde{\alpha}_{FBE,E^*} = 0$

consumption risk sharing. To understand the intuition behind the above optimal portfolios, we need to find out how the economy responds to various shocks in autarky.

As all prices are pre-set and output is demand-determined, productivity shocks have no effect on firm revenue. The only changes occur are associated with the allocation between labor income and profit. For example, if home firms experience a positive productivity shock, they need less labor to produce the same quantity of goods. Labor income decreases, but firm profit increases at the same time. If home agents hold one hundred percent of their own firms, their income as well as consumption are unaffected. In other words, the default equity position, or a complete home bias in equities, provides a perfect hedge against the productivity shocks. Hence, money supply shocks are the only source of income uncertainties left in the economy.

FBE Economy

With pre-fixed goods prices, the terms of trade move in the opposite direction of the exchange rate up to first-order. The excess return of foreign bonds relative to home bonds can be expressed as $\hat{r}_{B^*} - \hat{r}_B = -\hat{\tau} + O(\epsilon^2)$. Households can fully insure themselves against the money supply shocks by holding the right amount of nominal bonds ($\tilde{\alpha}_{FBE,B^*} = \frac{1-\theta}{2}$). For instance, if home experiences a positive money supply shock, home currency would depreciate, which causes a deterioration in home terms of trade. Home real GDP increases when $\theta > 1$. In this case, home households should lend in the home currency denominated bond and borrow in the foreign currency denominated bond ($\tilde{\alpha}_{FBE,B^*} < 0$). The gain in terms of real production income is balanced by a negative payment from bond holdings (when home currency depreciates). Therefore, home and foreign nominal bonds provide a perfect hedge against the money supply shocks. The financial market is complete as long as nominal bonds are allowed for trade.

Here, we confirm the Engel and Matsumoto (2005) result that in the presence of sticky prices, the returns to workers and those to firm owners become negatively correlated in response to the productivity shocks, leading to a home bias in investors' portfolios. However, this result also relies on the fact that consumption risk sharing is perfect. It may no longer be true when the financial market is incomplete.

FE Economy

Generally speaking, home and foreign equities alone cannot fully hedge against the productivity and the money supply shocks at the same time. The portfolio decisions become a lot more complicated as only two individual assets are available to deal with four competing hedging tasks. The exact equity holdings depend on the relative size between the two types of shocks and on the value of structural parameters.

Let the share of labor income equal to two thirds ($\zeta = \frac{2}{3}$) and the shocks be one percent of their steady state level ($\sigma_A^2 = \sigma_M^2 = 0.0001$). Figure 1 shows five subsets of the share of domestically owned home equities (δ_E) when the disutility of labor is assumed to be linear ($\psi = 0$) and both the relative risk aversion coefficient (ρ) and the elasticity of substitution between home and foreign goods (θ) are set to be within the range of $[0, 5]$.

The first thing to note is that when $\theta > 1$, home households take a long position in domestic equities ($\delta_E > 1$). Recall that productivity shocks are hedged perfectly if households hold one hundred percent of their domestic equities. Therefore, how to hedge against the money supply shocks using home and foreign equities in the FE economy determines the equilibrium equity positions.

In response to a positive home money supply shock, home currency depreciates. Demand for home goods goes up. The demand for labor and the wage rate increase at home, so does the firm's revenue when $\theta > 1$. As the majority of total income is paid to labor, the increase in labor cost is so significant that the dividend moves in the opposite direction to the firm's revenue. Hence, it is optimal for home households to aggressively invest in their domestic equities ($\delta_E > 1$) or take a short position in foreign equities ($\tilde{\alpha}_{FE,E^*} < 0$). In this way, the gain in terms of real production income is balanced by a negative payment from equity holdings.

Now let us look at the case when $\theta < 1$. Again, suppose the home country experiences a positive money supply shock, home currency depreciates. Home goods become relatively cheaper than foreign goods. Demand for home goods goes up, and so do the demand for labor and the wage rate at home. If $\theta < 1$, the expenditure switching effect is weak relative to the change in price. Thus, home real GDP decreases. The dividend on home equities drops too. It is optimal for home households to invest in foreign equities. Moreover, the smaller the expenditure switching effect (θ) is, the higher the share of foreign equities in home households' portfolios.

Figure 2 repeats the above exercise, but assumes the disutility of labor is in a quadratic form ($\psi = 1$). It is easy to see that the results are not sensitive to this modification.

The elasticity of substitution between home and foreign goods (θ) is a key parameter that determines the equilibrium equity holdings. The literature of international economics offer a wide range of estimations, from 1.2 to 21.4, as reviewed by Obstfeld and Rogoff (2000b). The number often used in the macroeconomics study is between 1 and 2, following Backus et al. (1994) and Chari et al. (2002). A value smaller than unity is seldom used. Stockman and Tesar (1995) set the elasticity of substitution between traded and non-traded goods equal to 0.44. Heathcote and Perri (2002) set the elasticity of substitution between home and foreign intermediate goods equal to 0.9. Let $\zeta = \frac{2}{3}$, $\sigma_A^2 = \sigma_M^2 = 0.0001$ and $\rho = 1.5$, Table 3 describes the equilibrium share of domestically owned home equities in the FE economy for different values of θ and ψ.

Table 3: Share of Domestically Owned Home Equities with Sticky Prices and Flexible Wages

$\delta_{FE,E}$	$\theta = 0.44$	$\theta = 0.9$	$\theta = 1.5$
$\psi = 0$	0.7445	0.9624	**1.1957**
$\psi = 1$	0.8284	0.9805	**1.0818**

Based on the most typical calibration in the macroeconomics literature, the optimal portfolios involve a substantial short position in foreign equities (see the last column in Table 3), because the labor income and the return to domestic equities are negatively correlated with respect to both the productivity and the money supply shocks. This "super home bias" result casts doubt on the ability of sticky price alone in characterizing the observed equity portfolios.

Wage adjustments are likely to have strong implications for the correlation between the returns to workers and the returns to firm owners. In fact, there are more extensive empirical evidence for sluggish wages than that for sticky prices.[9] We are going to examine the effect of nominal wage rigidity in the next section.

4 A Model with Sticky Wage

Following Obstfeld and Rogoff (2000a), each worker is a monopolistic supplier of a distinctive variety of labor services. Workers set wages (in their domestic currency) before shocks are realized.[10] Each firm uses workers of every type and the elasticity of substitution among varieties of labor is given by μ. The other parts of the model are kept same as before.

[9]See, for example, Altonji and Devereux (2000), Fehr and Goette (2005), and Goette et al. (2007).

[10]Because the real marginal consumption value of wage is not equal to the marginal disutility of working when wages are pre-set, only sufficiently small shocks are considered here to avoid a further discussion of voluntary participation constraint. This concern also applies to sticky goods prices, as firms may have to operate under negative profits if a big negative productivity shock hits the economy.

The production function for aggregate home firms is

$$Y_H = AL = A\left[2^{\frac{1}{\mu}}\int_0^{\frac{1}{2}} l(z)^{\frac{\mu-1}{\mu}}dz\right]^{\frac{\mu}{\mu-1}} \tag{35}$$

which implies that their demand for agent z's labor is

$$l(z) = \left[\frac{w(z)}{w}\right]^{-\mu} L \tag{36}$$

Before shocks are realized, agent z sets his wage to maximize his expected utility

$$E\left[U(c(z), \frac{m(z)}{P}, l(z))\right]$$

subject to

$$m(z) + Pc(z) = m_0(z) + w(z)l(z) + \pi(z) + P\sum_{k=1}^{n}\alpha_k(z)r_k + t(z)$$

In a symmetric equilibrium, all agents will choose the same wage rate

$$w = \frac{\eta\mu}{\mu-1}\frac{E\left[L^{1+\psi}\right]}{E\left[\frac{L}{PC^{\rho}}\right]} \tag{37}$$

The wage equation (37) takes the place of the intratemporal labor-leisure choice equation (16) in the previous set of equilibrium conditions. Table 4 describes the optimal portfolio holdings in the FE and FBE economies in the presence of nominal wage rigidities.

Sticky Wage and Flexible Price Case

When wages are fixed but goods prices are flexible, the excess return of foreign equities

Table 4: Optimal Portfolio Holdings with Sticky Wages

$\kappa = 0:$	
FE	$\tilde{\alpha}_{FE,E^*} = \frac{1}{2}$
FBE	$\tilde{\alpha}_{FBE,B^*} = 0$
	$\tilde{\alpha}_{FBE,E^*} = \frac{1}{2}$
$\kappa = 1:$	
FE	$\tilde{\alpha}_{FE,E^*} = \frac{1}{2}\frac{(\theta-1)^2(\zeta-1)^2\sigma_M^2}{\zeta^2[1+\rho(\theta-1)]\sigma_A^2+(\theta-1)^2(\zeta-1)^2\sigma_M^2}$
FBE	$\tilde{\alpha}_{FBE,B^*} = \frac{1-\theta}{2}$
	$\tilde{\alpha}_{FBE,E^*} = 0$

relative to home equities is $\hat{r}_{E^*} - \hat{r}_E = (\theta - 1)\hat{\tau} + O(\epsilon^2)$.[11] Hence, the portfolio decisions are identical to the case where both wages and prices are flexible. Consumption risk sharing is perfect so long as home and foreign equities are allowed for trade.[12]

Sticky Wage and Sticky Price Case

When goods prices are sticky, the excess return of foreign bonds relative to home bonds becomes $\hat{r}_{B^*} - \hat{r}_B = -\hat{\tau} + O(\epsilon^2)$. Thus, the presence of nominal wage rigidity has no effect on the optimal portfolios in the FBE economy. Households will still hold one hundred percent of their domestic equities to hedge against the productivity shocks, and borrow (lend) in foreign (home) nominal bonds to hedge against the money supply shocks when $\theta > 1$.

In the FE economy, home and foreign equities cannot simultaneously hedge against the productivity shocks and the money supply shocks. The consumption risk sharing is imperfect in this case. Once again, let $\zeta = \frac{2}{3}$, $\sigma_A^2 = \sigma_M^2 = 0.0001$ and $\rho = 1.5$, Table 5 describes the

[11] Detailed derivation see Appendix A.3

[12] One thing different here is that money has real effect on the economy now. In response to a positive home money supply shock, home currency depreciates but home goods prices are not adjusted due to fixed wage rates. Home terms of trade deteriorate and home real GDP becomes relatively higher than foreign real GDP. Although the equilibrium asset positions are not affected by this additional friction, the welfare changes. In fact, the welfare is lower because the two countries produce at different levels when they are equally productive.

equilibrium share of domestically owned home equities for different values of θ.[13]

Table 5: Share of Domestically Owned Home Equities with Sticky Prices and Sticky Wages

	$\theta = 0.44$	$\theta = 0.9$	$\theta = 1.5$
$\delta_{FE,E}$	0.5067	0.9956	**0.9483**

For $\theta > 1$, we no longer have the "super home bias"result as in the case with flexible wages. With fixed goods prices, households' default equity positions provide a perfect hedge against the productivity shocks. Again, what is important is how home and foreign equities are used to hedge against the money supply shocks. In response to a positive home money supply shock, home currency depreciates. Demand for home goods goes up, so do the demand for labor and the firm's revenue (when $\theta > 1$). Remember that nominal wages are fixed. Even the labor income accounts for two thirds of the total income, the increase in labor cost is not big enough to reduce home firms' profits. The dividend on home equities is positively correlated with the labor income. It is optimal for home households to diversify their portfolios and invest in foreign equities ($\tilde{\alpha}_{FE,E^*} > 0$). In this way, the gain in terms of real production income is balanced by a negative payment from equity holdings.

Figure 3 shows five subsets of the share of domestically owned home equities (δ_E) when both the relative risk aversion coefficient (ρ) and the elasticity of substitution between home and foreign goods (θ) are set to be within the range of $[0, 5]$. For most values of $\theta \in [0.8, 2]$, the model with sticky wages and sticky goods prices infers optimal equity portfolios that match the data quite well. With incomplete financial markets, the model predicts home bias for a wide range of parameterization, especially when the cut-off value for home bias is set to be 0.75 in stead of 0.9 (see Figure 4).

[13]As wages are pre-set, ψ has no effect on the portfolio choices.

5 Conclusion

This paper analyzes the optimal portfolio decisions in a monetary open economy framework with particular attention to the completeness of financial markets and the presence of nominal rigidities in goods prices and in wages.

With complete financial markets and sticky prices, the model generates a complete home bias in equities because the return to human capital and the return to domestic firms are negatively correlated with respect to the productivity shocks. However, this result is sensitive to the configuration of financial markets. With incomplete financial markets, the model produces a "super home bias"result, as the return to human capital and the return to domestic firms are negatively correlated with respect to not only the productivity shocks but also the monetary shocks. The optimal portfolios actually require households to take a short position in foreign equities rather than diversify their asset holdings. Therefore, sticky prices alone cannot fully explain the observed portfolios.

Introducing sticky wages help match the data because the return to human capital and the return to domestic firms become positively correlated with respect to monetary shocks in this case. In fact, the model with incomplete financial markets and with nominal rigidities in both goods prices and wages predicts equity home bias for a wide range of parameterization used in the macroeconomics literature.

An interesting direction for future research is to conduct welfare analysis for models incorporating endogenous portfolio choices, especially those with incomplete financial markets. Portfolio decisions can be very complicated as a limited set of assets often need to deal with many competing hedging tasks in these environments. Such models may have intriguing policy implications. After all, the real world is far from having perfect risk sharing.

References

Altonji, J., and P. Devereux (2000) 'The extent and consequences of downward nominal wage rigidity.' *Research in labor economics* 19, 383–431

Backus, D.K., P.J. Kehoe, and F.E. Kydland (1994) 'Dynamics of the trade balance and the terms of trade: The j-curve?' *American Economic Review* 84(1), 84–103

Baxter, M., and U.J. Jermann (1997) 'The international diversification puzzle is worse than you think.' *American Economic Review* 87, 170–180

Bottazzi, L., P. Pesenti, and E. van Wincoop (1996) 'Wages, profits and the international portfolio puzzle.' *European Economic Review* 40, 219–254

Chari, V.V., P.J. Kehoe, and E.R. McGrattan (2002) 'Can sticky price models generate volatile and persistent real exchange rates?' Federal Reserve Bank of Minneapolis Research Department Staff Report 277

Devereux, M., and A. Sutherland (2006) 'Solving for country portfolios in open economy macro models.' Manuscript, University of British Columbia and University of St. Andrews

Engel, C., and A. Matsumoto (2005) 'Portfolio choice in a monetary open-economy dsge model.' Manuscript, University of Wisconsin and IMF

Evan, M., and V. Hnatkovska (2005) 'A method for solving general equilibrium models with incomplete markets and many financial assets.' Manuscript, Georgetown University

Fehr, E., and L. Goette (2005) 'Robustness and real consequences of nominal wage rigidity.' *Journal of Monetary Economics* 52(4), 779–804

Francis, N., and V.A. Ramey (2005a) 'Is the technology-driven real business cycle hypothesis dead? shocks and aggregate fluctuations revisited.' *Journal of Monetary Economics* 52 (8), 1379–1399

_ (2005b) 'Measures of per capita hours and their implications for the technology-hour debate.' NBER Working Paper, No. 11694

French, K.R., and J.M. Poterba (1991) 'International diversification and international equity markets.' *American Economic Review* 81(2), 222–226

Gali, J. (1999) 'Technology, employment, and the business cycle: Do technology shocks explain aggregate fluctuations?' *American Economic Review* 89, 249–271

Gali, J., and P. Rabanal (2004) 'Technology shocks and aggregate fluctuations: How well does the rbc model fit postwar u.s. data?' NBER Macroeconomics Annual

Goette, L., U. Sunde, and T. Bauer (2007) 'Wage rigidity: Measurement, causes, and consequences.' *Economic Journal* 117, 499–507

Heathcote, J., and F. Perri (2002) 'Financial autarky and international business cycles.' *Journal of Monetary Economics* 49, 601–627

_ (2007) 'The international diversification puzzle is not as bad as you think.' Minnesota Economics Research Report No. 2007-03

Hnatkovska, V. (2005) 'Home bias and high turnover: Dynamic portfolio choice with incomplete markets.' Manuscript, Georgetown University

Jermann, U. (2002) 'International diversification and endogenous labor supply choice.' *European Economic Review* 46, 507–522

Kollmann, R. (2005) 'International portfolio equilibrium and the current account: A dynamic general equilibrium perspective.' Manuscript, University of Paris XII

Kraay, A., N. Loayza, L. Serven, and J. Ventura (2005) 'Country portfolios.' *Journal of European Economic Association* 3(4), 914–945

Lewis, K.K. (1999) 'Trying to explain home bias in equities and consumption.' *Journal of Economic Literature* 37(2), 571–608

Lucas, R.E. Jr (1982) 'Interest rates and currency prices in a two-country world.' *Journal of Monetary Economics* 10, 335–359

Obstfeld, M., and K. Rogoff (2000a) 'New direction for stochastic open economy models.' *Journal of International Economics* 50, 117–153

_ (2000b) 'The six puzzles in international macroeconomics: Is there a common cause?' NBER Working Paper No. 7777

Rotemberg, J. (2003) 'Stochastic technical progress, smooth trends and nearly distinct business cycles?' *American Economic Review* 93 (5), 1543–1559

Stockman, A.C., and L.L. Tesar (1995) 'Tastes and technology in a two-country model of the business cycle: Explaining international comovements.' *American Economic Review* 85(1, 168–185

Tesar, L.L., and I. Werner (1995) 'Home bias and high turnover.' *Journal of International Money and Finance* 14(4), 467–492

Figure 1: Share of Domestically Owned Home Equities with Flexible Wage and Linear Disutility in Labor

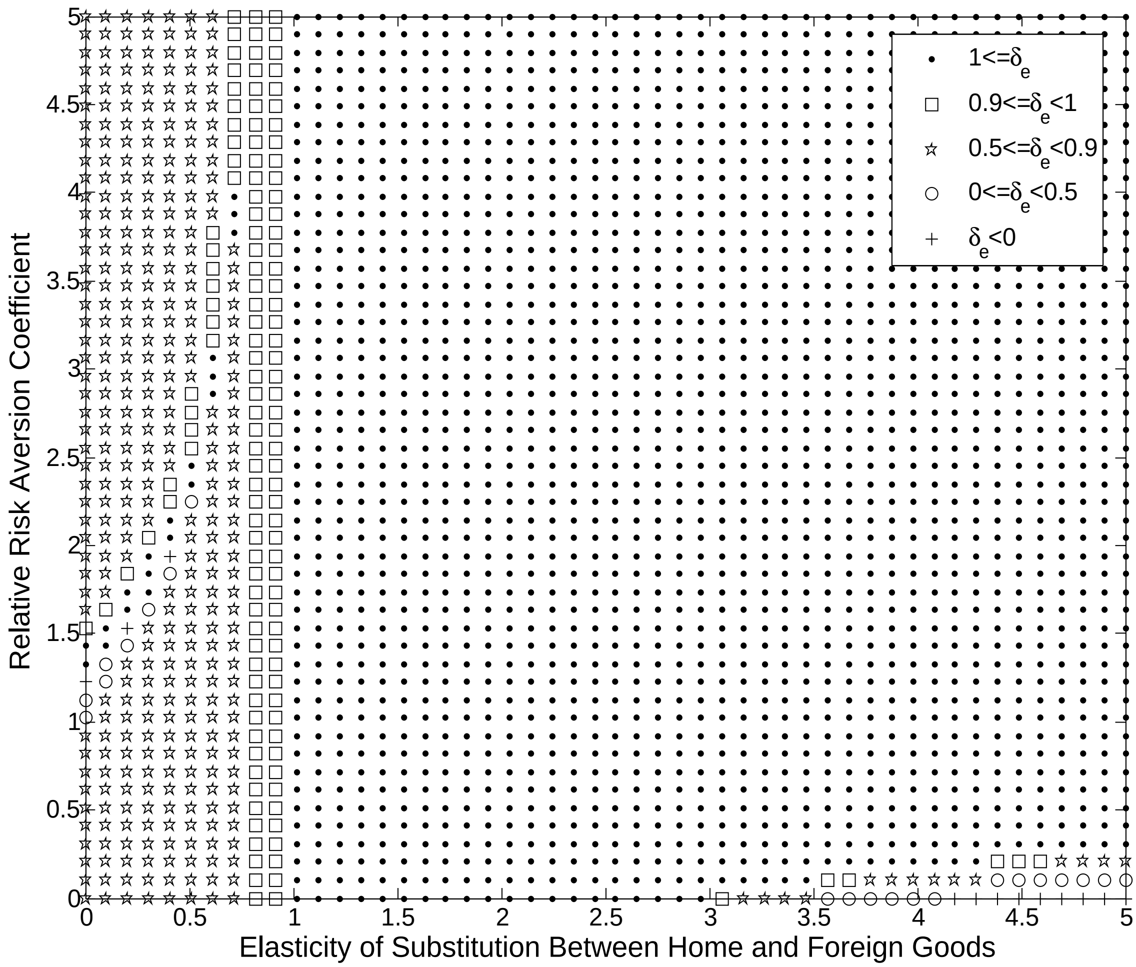

Figure 2: Share of Domestically Owned Home Equities with Flexible Wage and Quadratic Disutility in Labor

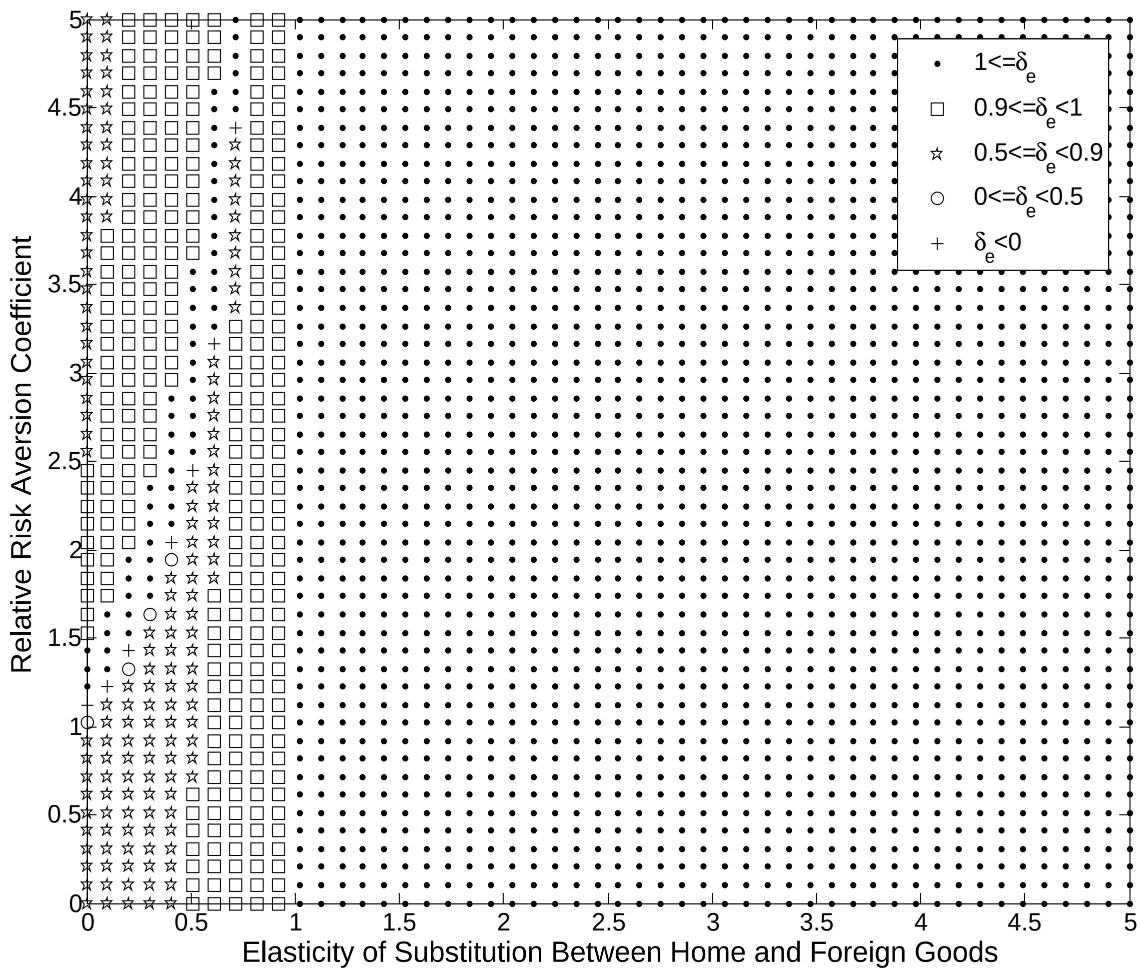

Figure 3: Share of Domestically Owned Home Equities with Sticky Wage and Cut-off Value 0.9

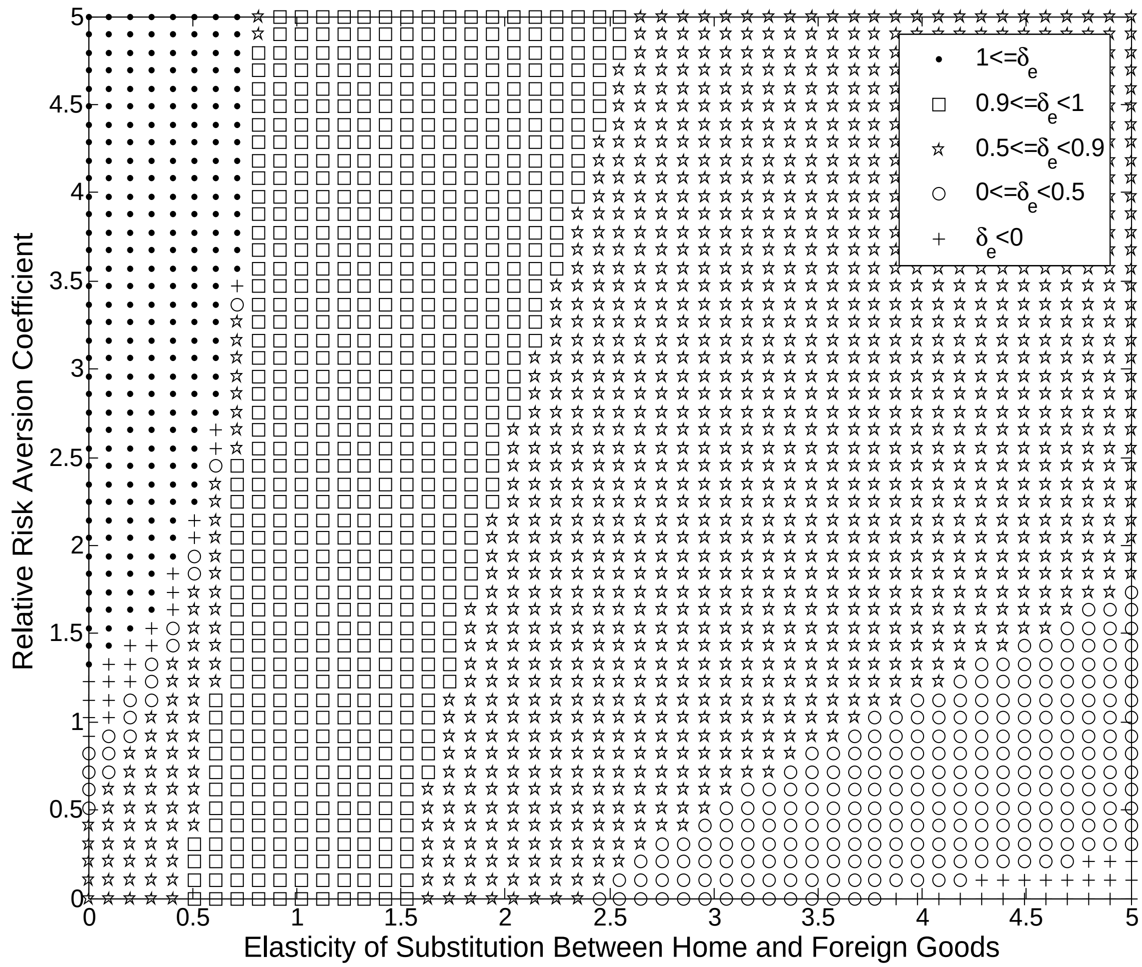

Figure 4: Share of Domestically Owned Home Equities with Sticky Wage and Cut-off Value 0.75