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Misallocation of Resources, Political Connections and External Flows

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Abstract

This paper shows that the current design of foreign aid and loans may impede growth in developing economies with weak political institutions. First, the paper provides empirical evidence that politically connected Pakistani firms pay lower effective taxes and this tax differential increases with the public external debt to GDP ratio. I then develop a political economy model in which agents connected with the government receive lower taxes and barriers to entry in exchange for political support, causing misallocation in the economy. High external flows give the government more room to lower taxes on connected entrepreneurs, which keeps low productivity, connected firms in the market. I calibrate the model to the economy of Pakistan and show that reducing flows by 30% reduces inequality and generates an output gain of 12%. I also show that a similar outcome could be obtained by adding conditions to existing external flows that require a higher level of fiscal revenues or that reduce barriers to entrepreneurship.

JEL Classification: E60, EO10, P16

Keywords: Corporate taxes, Economic development, External debt, Fiscal revenues, Foreign aid, Misallocation of resources, Political connections, Political economy.

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1 Introduction

One of the main purposes of foreign aid and external debt is to help developing countries that lack funds to achieve economic growth and reduce inequality. However, their effectiveness depends on factors such as proper governance, accountability, institutional and political environment of the recipient countries' and their priorities. Therefore, the impact of foreign aid and external debt on the economic growth of these economies is not straight forward. Some countries, such as South Korea, which relied heavily on foreign aid and external debt from the 1960s to 1980s, experienced significant and unparalleled economic growth (Ugwuegbe et al., 2016). In contrast, other countries, such as Pakistan, have struggled to achieve the desired economic growth, while their dependency on external debt to meet fiscal needs has increased substantially (Hussain et al., 2017). This raises three important questions in development economics: Why do some developing countries experience rapid economic growth while others experience slow growth, despite receiving high foreign aid and external debt? Additionally, do international aid and debt hinder economic growth in economies with weak political structures? And if it does, through which mechanism? There is no consensus on how to answer these questions in the literature.

Meanwhile, evidence indicates that in countries with weak institutions, resource allocation is often driven by political connections rather than economic potential, resulting in the persistence of less productive firms. These institutions frequently fail to enforce merit-based distribution, allowing political elites to allocate resources based on personal ties, loyalty, or political favors, rather than economic viability. This system of political patronage distorts competition, channeling resources to politically connected individuals or entities regardless of their productivity. This paper builds on these observations by developing a model that demonstrates how external loans and foreign aid can reinforce such patronage systems, where benefits are exchanged for elite support, ultimately perpetuating resource misallocation and impeding economic growth.

This paper has two main objectives: first, to empirically examine the relationship between preferential treatment of connected firms and external flows; second, to develop a theoretical model demonstrating how this interaction may lead to resource misallocation, using Pakistan as a case study. To the best of my knowledge, this paper is the first attempt in the literature to develop and simulate a theoretical political economy model to rationalize the connection between the preferential treatment received by the connected firms and the external debt and foreign aid. It also shows that in the presence of higher external flows, evolution of entrepreneurial skill and the measure of highly productive entrepreneurs in the economy are crucial in determining the extent of misallocation. The results from simulation of the model are than supported by the empirical analysis which finds a significant evidence of a positive correlation between the public external debt and the lower effective tax rates paid by the politically connected firms in Pakistan.

The empirical analysis uses firm-level and election data from Pakistan. Using this data, I find significant evidence of politically connected firms paying 8.37 percentage points lower effective tax rates than non-connected firms. I also find that a 10 percentage points increase in the public external debt to GDP ratio, increases this tax differential and further lowers the effective tax rate paid by the politically connected firms by 9.26 percentage points.

The theoretical analysis proposes a model where the entrepreneurial skills of the individuals evolve overtime. In the model the patronage system consists of preferential treatment received by politically connected firms. This preferential treatment is in terms of paying lower effective tax rates and bypassing the existing barriers to entrepreneurship. This creates misallocation of resources by encouraging low productivity, politically connected firms to produce in the economy. In return for this privilege, the Elite increases its ability to remain in power. Thus, by manipulating tax rates on the connected, government can choose between high tax - no misallocation and low tax - misallocation equilibrium. In this environment, higher foreign aid and external debt flows allow the government to keep the taxes on connected entrepreneurs low which, in turn, can sustain a significant fraction of low-productivity connected firms in operation. In particular, the dynamics of the model show that overtime if the measure of low skilled individuals increase, higher foreign aid and external debt (external flows) worsens the output of the economy. This results in an equilibrium with misallocation and lower output.

I calibrate the model to the economy of Pakistan. In the baseline calibration, Pakistan is in an equilibrium with misallocation. Counterfactual simulations show that reducing external flows by 30% results in an equilibrium without misallocation, a 34% reduction in inequality measured by the Gini coefficient, a 32.4% increase in the welfare of the non-connected entrepreneurs and a 12% increase in the steady state output. I also show that a similar outcome is possible by keeping the debt at the same level but making it contingent on achieving a certain level of direct tax revenues or reducing entry barriers to entrepreneurship. The outcomes suggest the need to restructure existing foreign aid and debt packages in three possible directions: (1) reducing the level of unconditional debt and aid or (2) keeping the same level of debt and aid but adding conditions that require the government to either keep a pre-determined level of tax revenue or (3) to reduce entry barriers to entrepreneurship. The first two options cause the Elite to increase tax rates on the connected entrepreneurs to raise revenues and make it unprofitable for low skilled, connected, firms to stay in the market. The third option reduces the entry costs for non-connected, highly skilled entrepreneurs, making it more profitable for them to enter the market and produce.

The remainder of the paper is organized as follows. Section 2 reviews the current literature related to this paper. Section 3 presents the data, methodology and results of the empirical analysis. Section 4 develops a model of misallocation consistent with the empirical findings. Section 5 discusses the calibration of the model. Section 6 presents the simulation results of the baseline model using the calibrated parameters, Section 7 performs counterfactual exercises, Section 8 provides the results from the sensitivity analysis, Section 9 discusses policy recommendations based on the results of the model, and Section 10 concludes.

2 Literature Review

This paper contributes to three areas of the literature. First, it adds to studies investigating the relationship between foreign aid, external debt, growth, and institutional quality. Despite extensive empirical work, there remains little consensus on the impact of external flows on growth. Some studies using cross-sectional and panel data for multiple countries find little to no robust evidence of the effectiveness of foreign debt and aid on growth (Rajan & Subramanian, 2007; Reinhart & Rogoff, 2010). Reinhart & Rogoff (2011) find that high levels of external debt often precede banking and sovereign debt crises. Other studies suggest that external flows may reduce growth in some developing economies, with high debt levels contributing to debt-overhang problems, reduced investment, and lower GDP per capita (Akram, 2011; Choong et al., 2010; Clements et al., 2003; Guei, 2019). Chatterjee et al. (2012) develop a model demonstrating that foreign aid can be fungible, relaxing the government's budget constraint, and reducing public goods spending. They also find, using panel data from 67 countries, that 70% of foreign aid is fungible and, in the presence of corruption and rent seeking, aid is ineffective in promoting growth. In contrast, other studies find that aid positively influences growth when accompanied by sound fiscal and trade policies (Burnside & Dollar, 2000).

There is strong empirical evidence linking institutional quality to economic growth, and some studies indicate that foreign aid can have a detrimental effect on institutional quality (Acemoglu et al., 2005; Young & Sheehan, 2014). Acemoglu (2008) develop a closed-economy model comparing the economic performance of oligarchic and democratic regimes. They show that, in the long run, oligarchic institutions are less efficient, exhibit greater income inequality, and have lower aggregate output than democratic regimes, which set lower barriers to entrepreneurial entry. In oligarchies, low-productivity firms persist, leading to inefficient resource allocation. This paper builds on Acemoglu (2008) by extending the model to include external flows and political patronage. Unlike Acemoglu (2008), the model in this paper assumes that policy decisions are made by an independent elite, and agents differ based on their political connections to the elite, with entrepreneurship decisions endogenously determined by the policies set by the elite.

This paper also relates to the literature on resource misallocation, which links misallocation to total factor productivity (TFP), capital accumulation, and growth. Restuccia & Rogerson (2008) develop a growth model with firm-level heterogeneity, showing that preferential policies can lead to significant capital misallocation and reduce aggregate TFP. Hsieh & Klenow (2009) use microdata from China and India to quantify misallocation, finding that reallocating capital and labor to equalize marginal products with the United States would yield TFP gains of 30-50% for China and 40-60% for India. Huneeus & Kim (2018) show that reducing lobbying activities would increase U.S. aggregate productivity by 6%. Fattal-Jaef (2022) find that high entry barriers for firms distort allocative efficiency, with their removal potentially leading to 8% productivity gains. None of these studies, however, consider external flows as a source of misallocation. The model developed in this thesis shows that reducing external flows in Pakistan leads to gains in aggregate productivity and output by eliminating misallocation and reallocating capital and labor to more productive firms.

The empirical section of this paper also relates to literature on firms using political connections to gain preferential treatment, such as credit access at lower interest rates or reduced taxes, irrespective of productivity (Ashraf et al., 2020; Khwaja & Mian, 2005; Saeed et al., 2019). Faccio (2006) analyze data from 47 countries and find that politically connected firms have higher leverage, larger market shares, and lower performance compared to non-connected firms, with the effects being more pronounced in less developed countries with high corruption. Khwaja & Mian (2005) provide evidence from Pakistan showing that politically connected firms receive more credit and have higher default rates than non-connected firms, largely due to government bank lending practices. Akcigit et al. (2023) develop a growth model showing that politically connected firms have lower innovation and productivity. Empirical evidence from Italy supports their model, demonstrating that while politically connected firms have higher survival rates and revenues, the aggregate losses from misallocation and reduced growth outweigh any gains from preferential policies.

To the best of my knowledge, the literature has not yet explored the link between political connections and external flows such as foreign aid and debt. This paper fills the existing gap by examining the intersection of external flows, political connections, and misallocation. Specifically, it shows that the effectiveness of external aid and debt depends on the degree of privileges granted to politically connected individuals. Furthermore, this paper demonstrates that external flows may exacerbate preferential treatment, perpetuating or even intensifying misallocation of resources toward low-productivity, politically connected entrepreneurs.

In the next section, using data from Pakistan and Pakistani firms, I estimate the impact of a firm being politically connected on their effective tax rates and the impact of the external debt on the effective tax rates paid by the politically connected firms.

3 Empirical Analysis

This section describes the database, the econometric methodology and presents the results from the empirical analysis. The aim of this econometric analysis is two fold. First, to find a relationship between effective tax rates and the political connectivity of a firm. Second, to establish the relationship between the external debt and the effective tax rates paid by the politically connected firms. I first discuss the dataset related to Pakistan, used to obtain the variables for this analysis (subsection 3.1) and describe the construction of the main variables (subsection 3.2). Then, I describe the econometric methodology used for the estimation (subsection 3.3), followed by the discussion of the main results (subsection 3.4).

3.1 Data sources

The data for the period of 2013-2019 used in this analysis is collected from three main sources. The financial variables and information on the board of directors of the Pakistani firms is taken from the S&P capital IQ database (S&P Capital IQ, 2021). The data for the external debt , GDP and the macro variables used in the model is taken from the World Bank, specifically the external debt related information is taken from the international debt statistics (IDS) country tables for Pakistan (World Bank, 2021). The database of candidates who participated in the 2013 and 2018 general election is constructed using the documents and list available from the Election Commission Pakistan (ECP) (Election Commission of Pakistan, 2021). The missing data on the financial variables of the firms from the S&P platform is also supplemented from the summary of the annual reports obtained from the State Bank of Pakistan (SPB) and in some cases hand collected from the individual annual reports of each company available on their websites or other sources on the Internet (State Bank of Pakistan, 2021).

Sample Selection The empirical analysis in this section is conducted using a balanced panel sample of 268 listed non-financial Pakistani firms covering the period from 2012 to 2019. Additional details on the sample selection procedure are provided in Appendix A.1. Table 6 in the Appendix A.1 presents the distribution of the sample firms based on their connections.

3.2 Variables Measurement

3.2.1 Firm level: Micro variables

Political Connectedness Following the work of Fama & French (1992) and Khwaja & Mian (2005) I define a politically connected firm as a firm which has one or more members of its board of directors who are politically connected. A member is defined to be politically connected until the next general election if their full name (First, Middle, if applicable and Last) matches with the name of a candidate who took part in the general election.[1] To establish the database on connected board of directors, I use the information available on the key board of directors for the sample firms in the S & P capital IQ database and match it with the database containing the full names of candidates who took part in the general elections of 2013 and 2018 in Pakistan, constructed from the data available with the ECP (Election Commission of Pakistan, 2021). Some of the firms in the original sample had more than one politically connected board member who took part in a particular year's general election. According to the definition there is no difference between firms having one or multiple board members who are politically connected.

In the following analysis a firm's status of political connectedness is represented by a time-variant dummy variable POLCON, which takes value one if the firm has at least one politically connected person in its board of director and zero otherwise.

Measure of tax rates Following previous studies in the empirical finance and accounting literature (Adhikari et al., 2006; Gupta & Newberry, 1997), a firm's tax rate is measured by calculating the effective income tax rate (ETR). A firm's ETR is defined as tax expenses excluding any portion of deferred tax expenses which is not yet paid as a ratio of the profit or earnings before interest and tax (EBIT) (Faccio, 2006). An alternative would be to use operating cash flow as income instead of EBIT (Zimmerman, 1983). [2] More than 50% of firms in my sample had missing information on operating cash flow for certain years, which can not be supplemented from any other sources. Thus, I adopt the more common practice

[1] Here, I assume that if any board member of a previously connected firm is not a candidate taking part in the next general election the firm loses its connection, as it is possible that the member is not strongly affiliated with politics or a political party anymore and do not receive any preferential treatment.

[2] These studies do not show significant difference in their regression analysis obtained by using the operating cash flow compared to EBIT.

of using EBIT as the taxable income. The ETR in our sample is calculated as:

$$ETR = \frac{Tax\ expense - Deferred\ taxes}{EBIT}$$

There are some measurement issues which are related to the calculation of ETR as follows; 1) there are firms with negative taxes (tax refunds) 2) there are firms that have positive taxes and negative EBIT and 3) there are firms that have unreasonably small denominator (EBIT) resulting in ETR to be greater than 1 or tax rate to be higher than 100%. Following previous studies (Adhikari et al., 2006; Gupta & Newberry, 1997), I retain these firms in the sample and use the following data cleaning and recoding scheme: 1) set ETR = 0 for firms with tax refunds 2) set ETR =1 for firms with positive taxes and negative income and 3) constrain the ETR ratio of the sample to be between 0 and 1 so that the maximum tax rate is set at 1 for firms with ETR above 1.

Control Variables I control for the firm size using the natural logarithm of total assets (SIZE), for tangible assets using the ratio of fixed assets diveded by the total assets (COLLATERAL) and for firm's profitability or return on assets (ROA) by dividing the firm's EBIT by the total assets. These control variables have been identified as being correlated with firm performance and other financial decisions by empirical studies in finance like Adhikari et al. (2006), Faccio (2006) and Saeed et al. (2019).

3.2.2 Macro Variables

Public External Debt to GDP ratio The total external debt stock of Pakistan includes net loans owed to the IMF, long term public and publicly guaranteed external debt stock , long term private external debt stock, and short term external debt stock for a particular year end (World Bank, 2021). I use the public and publicly guaranteed external debt stock (which includes long term public and publicly guaranteed external debt stock and the short term external debt) and the nominal GDP of Pakistan in dollars to construct the (EDPGDP) ratio for the period of 2013-2019.[3]

Based on the premise of this study and observed facts, for politically connected firms, change in the preferential treatment in the form of lower taxes is more likely to be associated with the external debt obtained by the government compared to the external debt received by the private sector. Thus, I use (EDPGDP) for the main analysis. I also use the Total

[3]According to the world bank description of the short term external debt there is currently no accurate procedure to differentiate between private and public short term external debt.

External debt to GDP ratio (EDGDP) for robustness exercises, reported in the Appendix A.2.

Control Variables I include some macro level control variables which might be correlated with external debt to GDP ratio as well as the overall performance of the firms in the economy. I control for the government spending by using the government spending to GDP ratio (GOVGDP) (World Bank, 2021), lending interest rate measured using the annual average SBP lending interest rate (LIR) and for the foreign exchange effect using the average annual foreign exchange rate of Pakistani Rupee in terms of US dollars (FX) (State Bank of Pakistan, 2021).

Summary statistics for the final variables are provided in Table 7 in the Appendix A.1.

3.3 Model Estimation

Based on the above premises, I construct the following two hypotheses for the case of Pakistan:

Hypothesis 1: Ceteris Paribus, politically connected firms pay lower effective tax rates than non-connected firms.

Hypothesis 2: Ceteris Paribus, the effective tax rate differential between politically connected and non-connected firms is higher when the public external debt to GDP ratio is high.

I use individual fixed effects estimation technique for our model estimation. Fixed effects estimation is a popular method used for controlling endogenity caused by unobservable, firm specific, variables which might jointly determine financial variables and political connectivity of a firm (Adhikari et al., 2006; Saeed et al., 2019; Wintoki, 2007). The fixed effect model specification to test **Hypothesis 1** is as follows:

$$ETR_{it} = \alpha_0 + \beta_1 \cdot POLCON_{it} + \lambda \cdot X_{it} + \psi_i + \pi_t + \epsilon_{it} \tag{1}$$

where ETR is the effective tax rate of a firm, $POLCON$ is a dummy variable indicating political connectedness, it takes a value 1 if a firm is politically connected in a year, X_{it} are micro firm level control variables consisting of ($SIZE$), ($COLLATERAL$) and (ROA), ψ_i is individual firm fixed effects, π_t is time fixed effects and ϵ is the error term.

The fixed effects model specification to test **Hypothesis 2** are as follows:

$$\begin{aligned} ETR_{it} = \alpha_0 + \beta_1 \cdot POLCON_{it} + \beta_2 \cdot EDPGDP_t + \beta_3 \cdot POLCON_{it} * EDPGDP_t \\ + \lambda_1 \cdot X_{it} + \lambda_2 \cdot X_t + \psi_i + \epsilon_{it} \end{aligned} \tag{2}$$

where $EDPGDP$ is the public and publicly guaranteed debt stock as a ratio of the GDP and X_t are macro level control variables including ($GOVGDP$), (LIR) and (FX).

Our main coefficient of interest is β_1 for specification (1). The predicted sign for β_1 is negative which corresponds to politically connected firms paying lower effective tax rates than their non-connected peers. The main coefficient of interest for specification (2) is β_3 which captures the interaction between a firm being politically connected and EDPGDP on the effective tax paid by the connected firms. The predicted sign for β_3 is negative, if the politically connected firms pay lower effective taxes compared to the non-connected firms when EDPGDP increases.

3.4 The Effect of Political Connectedness and Public External Debt to GDP Ratio on Effective Tax Rates

Table 1 shows the regression results for specifications (1) and (2) using the sample of 268 firms. Column (1) of Table 1 shows that the coefficient of POLCON is positive and significant. More precisely the results show that politically connected firms pay 8.37 percentage points lower effective taxes than their non-connected peers, which supports **Hypothesis 1** and is in line with prior studies such as Adhikari et al. (2006) and Saeed et al. (2019). The control variables COLLATERAL and ROA are also significant, showing that as predicted these variables negatively affect the effective tax rates of a firm and is consistent with previous empirical studies (Adhikari et al., 2006; Faccio, 2006; Saeed et al., 2019).

Column (2) of Table 1 shows that the coefficient of the interaction term POLCON*EDGDP is significantly negative . This supports **Hypothesis 2** and shows that a 10 percentage points increase in the public and publicly guaranteed external debt to GDP ratio for Pakistan lowers the effective tax rates paid by the politically connected firms relative to the non politically connected firms by 9.26 percentage points. This supports our premise that the loans provided to the government of Pakistan result in an increase in the preferential treatment received in terms of lower taxes for the politically connected firms and might not be efficiently allocated to more productive use. Notice that column (2) shows that the coefficient of POLCON is significant but positive, contrary to our results for specification (1). This may not be completely counterintuitive as when an interaction term is added, the coefficient of the independent indicator term alone is not sufficient to measure the impact on the dependent variable. In this case the coefficient of POLCON shows the effect of POLCON on the effective tax rates when the public and publicly guaranteed external debt to GDP ratio is 0. However, as EDPGDP is a continuous variable and is unlikely to be 0 for any of the years, interpreting the coefficient of POLCON by itself in these specifications may not be informative for our analysis.

Table 9 in Appendix A.2 shows the results for specification (1) and (2) using the sample of 261 firms excluding the firms with the extrapolated financial variables as well as specification (2) using the ratio EDGDP instead of EDPGDP. It shows that results are not sensitive to the exclusion of the additional 7 firms. It also reinforces that the findings are consistent with the fact that the coefficient β_3 is higher when there is an increase in the ratio of EDPGDP compared to EDGDP which includes the private long term external debt.

To sum up, the empirical results presented in this section provides evidence of the existence of a significantly high measure of politically connected listed firms in Pakistan. It also confirms that one of the ways these firms receive preferential treatment due to their connectivity with the political parties is through paying lower effective tax rates than their non-connected peers and that this preferential treatment is higher when the external debt to GDP ratio increases.

In what follows, I develop a theoretical model that is consistent with the two previous facts and I use it to illustrate the main mechanism leading to misallocation under certain political environment. I then calibrate the model to Pakistan to obtain more quantitative results on the extent of the misallocation. Finally, I perform counterfactual experiments to determine the effects of changing the levels of external debt and foreign aid on misallocation and the total output of the economy.

Table 1: Main Regression Results

	(1) ETR	(2) ETR
POLCON	-0.0837**	0.168**
	(0.0339)	(0.0857)
EDPGDP		0.0858
		(0.747)
POLCON*EDPGDP		-0.926***
		(0.290)
SIZE	-0.00458	-0.0273
	(0.0390)	(0.0406)
COLLATERAL	-0.150**	-0.159**
	(0.0656)	(0.0639)
ROA	-0.293***	-0.270***
	(0.0651)	(0.0657)
GOVGDP		-4.885
		(6.600)
LIR		-1.200
		(0.807)
FX		0.00199
		(0.00205)
Firm fixed effects	Yes	Yes
Time fixed effects	Yes	No
No of Observations	1876	1876

Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Variable definitions: ETR = (Tax expenses- Deferred tax expenses)/(Earnings before interest rate and tax); POLCON=1 if the firm has a board of director who is politically connected; 0 otherwise; EDPGDP = (public and publicly guaranteed external long term debt stock+ short term external debt stock in US dollars)/ (Nominal GDP in US dollars); SIZE= Log of Total Assets; COLLATERAL= (Total Assets-Total Current Assets)/(Total Assets); ROA= (Earnings before interest and tax)/(Total Assets); GOVGDP= (Total government expenditure in US dollars)/(Nominal GDP in US dollars); LIR= Annual average SBP lending interest rate; FX= Average annual foreign exchange rate of Pakistani Rupee in terms of US dollars.

4 The Model

In this section I develop a dynamic political economy model with external flows that provides, the mechanism that is consistent with the empirical results in the previous section.

Subsection 4.1 describes the environment of the model and subsection 4.2 characterizes the economic equilibrium and provides the solution of the economic and political equilibrium.

4.1 Environment

I consider a dynamic political economy model in the spirit of Acemoglu (2008). Figure 1 provides an overview of the main components and the mechanism of the model. The model in this paper consists of an infinite horizon small open economy populated by one Elite who is in power and a continuum of risk neutral agents. In this economy agents are heterogeneous and they differ in three dimensions: political connectedness with the Elite, entrepreneurial ability and existing entrepreneurship status. Connected agents receive certain perks and patronage in return for their support of the Elite's ability to stay in power. These benefits constitute of (1) receiving a fixed transfer payment from the Elite, (2) being exempt from costly entry barriers into entrepreneurship and (3) receiving more favorable corporate taxation policies. Higher entry barriers to entrepreneurship for the non-connected agents can be interpreted as bribes taken by the public officials, the cost of bureaucratic procedures such as delays in getting licenses, permits and contracts, harassment by public officials, access to credit and other such forms of rent extraction (Desai et al., 2011). The barriers to entrepreneurship are constant and these entry barriers do not directly benefit the Elite. However, the politically connected agents have the privilege to bypass them when entering as entrepreneurs.

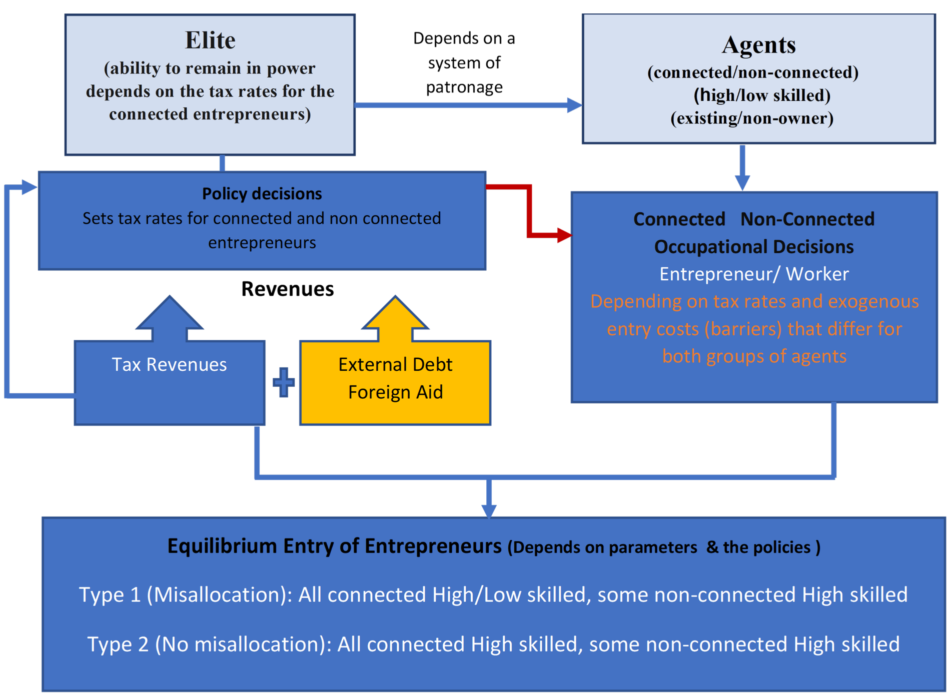


Figure 1: Model Framework

Every period the agents in the model make an occupational decision: they choose between becoming a worker or an entrepreneur. In the model only entrepreneurs have the ability to produce and the Elite makes all the policy decisions. I assume that only a small fraction of the agents have connections, so that in equilibrium some non-connected agents become entrepreneurs. Entrepreneurs face two type of taxes imposed by the Elite: a corporate tax and an entry barrier, the magnitude of both differs between connected and non-connected entrepreneurs.

Every period the Elite then chooses the tax rates for the entrepreneurs which maximize their own utility. It receives income in the form of tax revenues from the entrepreneurs and external flows, specifically external debt and foreign aid. The Elite's ability to remain in power is a function of the level of support from the connected agents, which is directly related to the tax rate set on connected entrepreneurs.

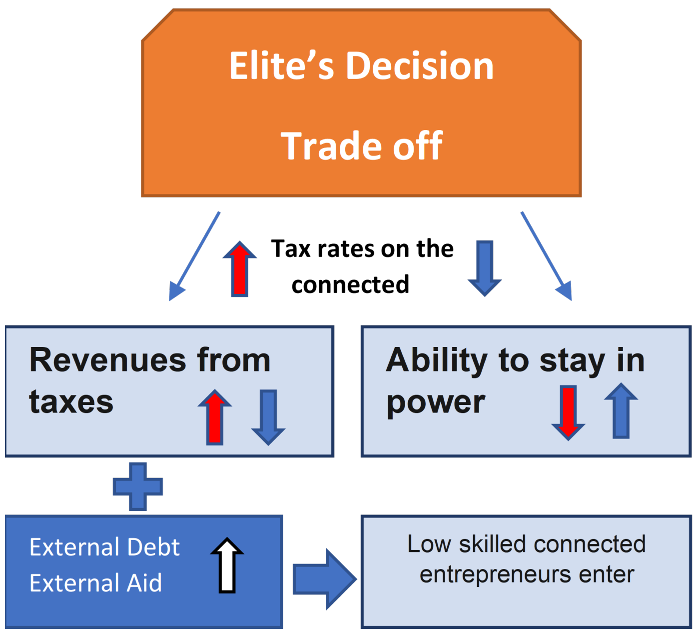


Figure 2: Elite's Trade off and External Flows

External flows which are exogenous and given in the model, have an important impact on the optimal tax rates set by the Elite. As depicted in Figure 2 in the absence of external flows the Elite set tax rates on the connected to optimize its trade off between revenues from the taxes and their ability to stay in power. High external flows provide more revenues and room for the Elite to set lower taxes on the connected entrepreneurs, simultaneously increasing their probability of staying in power. Lower taxes allow low skilled connected entrepreneurs to enter or remain in the market, crowding out the entry opportunity of the more productive high skilled non-connected entrepreneurs. The evolution of entrepreneurial skills over time alters the proportion of high-skilled agents and the measure of non-connected entrepreneurs in the economy. The model's transitional dynamics reveal that an increasing proportion of low-skilled entrepreneurs leads to worsening resource misallocation over time. This occurs as resources are inefficiently utilized by a larger share of low-productivity firms, negatively impacting the distribution of wealth, consumption, and potentially the overall output of the economy. In addition, the model also shows that lower level of external flows may increase the welfare of the non-connected entrepreneurs, as the measure of existing non-connected entrepreneurs who make positive profits are higher under an equilibrium where only high skilled individuals are entrepreneurs.

In what follows I explain the components of the model in more detail.

Agents

Distribution of agents There is a continuum of agents in the economy. An agent's type is determined by three characteristics: connectivity, entrepreneurial ability and existing entrepreneurship status. The connectivity of each agent is denoted by $j \in \{c, n\}$ as being with connections (c) or no connections (n). Entrepreneurial ability is determined by the level of skills, high (H) or low (L) and the ability is denoted by z. A^z is the productivity of an agent with ability $z \in \{H, L\}$ with $A^L < A^H$. Existing entrepreneurship status is given by $s \in \{e, r\}$, existing firm owner (e) no firm owner (r). The entrepreneurial ability and entrepreneurship status may change overtime, whereas connectivity is a permanent trait. Formally, I denote an agent's type at the start of the period t by m with $m = jzs$.

I assume that a fraction $\phi^c \in (0, 1)$ of agents are connected. Initially a fraction $M_0^H \in (0, 1]$ are high skilled agents. At the start of each period a fraction θ of entrepreneurs face an exogenous shock and exit from entrepreneurship, irrespective of their entrepreneurial ability or connectivity, and become workers. An individual's entrepreneurial ability z evolves over time following the process below.

$$z_{t+1} = \begin{cases} H & \text{with probability} \quad\quad \sigma^H \text{ if } z_t = H \\ H & \text{with probability} \quad\quad \sigma^L \text{ if } z_t = L \\ L & \text{with probability } 1 - \sigma^H \text{ if } z_t = H \\ L & \text{with probability } 1 - \sigma^L \text{ if } z_t = L \end{cases} \tag{3}$$

where $\sigma^H, \sigma^L \in (0, 1)$. Here σ^H is the probability that an agent is high skilled in period $t+1$ conditional on being high skilled in period t, and σ^L is the probability that an agent is high skilled in period $t+1$ conditional on being low skilled in period t. Following Acemoglu (2008) I assume that, $\sigma^H > \sigma^L > 0$ which means that the comparative advantage of entrepreneurs may change over time and that low skill levels do not become a permanent condition. This variability ensures that different individuals or entities are suited to entrepreneurship as the economic landscape changes. There are two interpretations of this dynamics: Firstly, entrepreneurial skills can fluctuate within individuals or across generations, requiring shifts in entrepreneurship as comparative advantages evolve. Secondly, individuals may possess fixed skills in various areas, with changes in comparative advantage prompting shifts in entrepreneurial activities. For instance, as industrial sectors become more profitable compared to agriculture, individuals with industrial expertise may enter entrepreneurship, while those with agricultural expertise may exit (Acemoglu, 2008).

Ownership status depends on the previous period's occupational choice. The transition for the firm ownership of an agent s_t evolves according to a simple rule as follows: agents jz who are entrepreneurs and operate a firm at time t and do not receive the exit shock are incumbent entrepreneurs at period $t+1$, $s_{t+1} = e$, otherwise they are workers at $t+1$, $s_{t+1} = r$. This implies that at the start of any given period a fraction of the total entrepreneurs in the previous period will endogenously lose their ownership of firm and become workers. These agents will then again have to pay the entry barriers again in order to enter as entrepreneurs in the current period.

For simplicity, as in Acemoglu (2008), I also assume that the initial fraction of existing entrepreneurs is zero.

With this notation, the initial distribution of agents among different types, N_0^m is then determined as follows:

(i) $N_0^{cHr} = \phi^c M_0^H$

(ii) $N_0^{cLr} = \phi^c(1 - M_0^H)$

(iii) $N_0^{nHr} = (1 - \phi^c)M_0^H$

(iv) $N_0^{nLr} = (1 - \phi^c)(1 - M_0^H)$

(v) $N_0^{cHe} = N_0^{cLe} = N_0^{nHe} = N_0^{nLe} = 0$

To focus the analysis on cases more pertinent to the purpose of the paper and aligning with real-world evidence, I introduce the following simplifying assumptions, which affect the distribution of agents:

Assumption 1. *It is the case that:*

(a) Connectivity and ability are independent, so the initial proportion of high skilled connected agents is $\phi^c M_0^H$.

(b) The size of the firm is fixed so that all firms hire the same measure of workers $\bar{L}$.

(c) The measure of connected agents cannot cover the whole market and satisfies $\phi^c < \frac{1}{\bar{L}}$.

(d) If all high skilled agents become entrepreneurs, they generate more than sufficient demand to employ the entire labor supply at any given period: $M_0^H \bar{L} > 1$ and $\bar{\sigma}\bar{L} > 1$, where $\bar{\sigma} = \frac{\sigma^L}{1-\sigma^L+\sigma^H}$.

(e) To avoid multiplicity of equilibria and convergence to a cycle in the long run, starting from any M_0^H, I restrict the analysis to cases where there are always some non-connected high skilled new entrepreneurs: the measure of firms dying every period satisfies $\phi^c + \sigma^H(1-\theta)(\frac{1}{\bar{L}} - \frac{1}{\bar{L}}\phi^c) < \frac{1}{\bar{L}}$ and the probability of a high skilled losing its skills is higher than of a low skilled gaining high skills $(1-\sigma^H) > \sigma^L$.

Assumption 1(a) implies that in equilibrium every period the measure of entrepreneurs in the economy is $\frac{1}{\bar{L}}$ and the assumption 1(b) implies that the total measure of connected agents is less than the equilibrium total measure of firms and, therefore, there are always some non-connected entrepreneurs in the economy. Assumption 1(d) ensures that the economy will not converge to a cycle in the long run, there will be no multiplicity of equilibrium and $(1-\sigma^H) > \sigma^L$ implies that there are always enough high skilled non-connected entrepreneurs to match the labour demand in equilibrium.

Distribution of high skilled agents

Let M_t denote the measure of high skilled agents in the economy at period t. In what follows I show that the evolution of M_t^H over time, is determined by exogenous parameters in the model. The transition rule for M_t^H is given by:

$$M_{t+1}^H = M_t^H - (1-\sigma^H)M_t^H + \sigma^L(1-M_t^H).$$

This can be simplified as $M_{t+1}^H = \sigma^H M_t^H + \sigma^L(1-M_t^H)$ and converges to a stationary point.

Lemma 4.1. *Let M_t^H be the measure of high skilled agents in period t. The sequence has a stationary point where $M^H = \bar{\sigma}$.*

Proof: Appendix A.3.

The next lemmas describe the evolution of M_t^H over time. They show that for any initial measure of high skill agents, the sequence converges monotonically to $\bar{\sigma}$.

Lemma 4.2. *Let M_0^H be an initial measure of high skilled agents at time* 0*. Then for all t, $M_t^H \in (0,1]$ and $M_0^H \underset{t\to\infty}{\to} \bar{\sigma}$.*

Proof: Appendix A.3.

Lemma 4.3. *The sequence M_t^H is monotone. In particular, (i) if $M_0^H < \bar{\sigma}$ then M_t^H is a strictly increasing sequence, (ii) if $M_0^H > \bar{\sigma}$ then M_t^H is a strictly decreasing sequence and (iii) if $M_0^H = \bar{\sigma}$ then M_t^H is a constant sequence.*

Proof: Appendix A.3.

The above lemmas show that the distribution of agents evolves overtime according to a exogenous process. This provides important results used to simplify the solution to the Elite's dynamic problem and solving the political equilibrium.

Agents' Decisions Let (B^c, B^n) be the exogenous and constant entry barriers for the connected and non-connected entrepreneurs. Let $p_t = (\tau_t^c, \tau_t^n)$ be the vector of policies at time t and let $p^t = \{p_n\}_{n=t}^{\infty}$ denote the sequence of policies from time t onward. Agents at time t make decisions taking the sequence of tax policies p^t and the sequence of wages $w^t = \{w_n\}_{n=t}^{\infty}$ and (B^c, B^n) as given. To simplify on notation and economize on space, I omit the dependence of choice variables on policies in what follows.

In this economy the expected lifetime utility of an agent of type m is then given by the following preferences:

$$U_0^m = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t c_t^m, \tag{4}$$

where $c_t^m \in \mathbb{R}$ denotes consumption of an agent of type m and $\beta < 1$ is the discount factor. The agents have risk neutral preferences and they do not exhibit risk averse behavior. They evaluate different options solely based on their expected monetary payoffs.

Each period an agent makes the occupational decision x_t on whether to work as an entrepreneur ($x_t = V$) or to be employed as a production worker ($x_t = W$). Entrepreneurs make an investment decision $k_t \geq 0$ and a labor input decision $\ell_t \geq 0$. Individuals who decide to become new entrepreneurs face costly entry barriers B^j. I normalize this cost to $B^c = 0$ for the connected entrepreneurs and assume $B^n > 0$ for the non-connected entrepreneurs. With this normalization, there is no difference between the problem and decisions of existing and new connected entrepreneurs. Hence forth, to save on notation I denote a particular type of connected entrepreneur as $m = cz$ and omit the entrepreneurship status indicator.

Technology Each period entrepreneurs produce a single non-storable final good denoted by y_t. As in Acemoglu (2008) the entrepreneur itself works as one of the production workers in the firm, which implies that the opportunity cost of becoming an entrepreneur is 0. An entrepreneur with skill level z can produce using the following production technology:

$$y_t = \frac{1}{1-\alpha}(A^z)^\alpha (k_t)^{1-\alpha}(\ell_t)^\alpha \tag{5}$$

where $\alpha \in (0,1)$ represents the income share of labor.

For simplicity also as in Acemoglu (2008), I allow for negative consumption if an entrepreneur wants to invest more than its output in a given period. This implies that the price of capital relative to output is equal to 1 in equilibrium. As mentioned in assumption 1 (b) the number of workers hired by each firm is fixed at $\bar{L}$, thus, $\ell_t = \bar{L}\ \forall\ t$.

Finally, I assume that each entrepreneur must operate their own firm and delegation to high skilled managers is prohibitively costly. So the entrepreneur's skill A^z matters for output. Without the latter assumption entry barriers would create no distortions. This is because if costly delegation is allowed low skilled entrepreneurs can then hire highly skilled managers increasing the productivity of their firms.

Elite

The Elite is a separate entity from the agents. The Elite is in power and sets the sequence of taxes p^t, with $\tau_t^c, \tau_t^n \in [0, \alpha]$. I assume the upper bound on taxes, to be equal to α, the labour share of income. I justify this upper bound by assuming that entrepreneurs can hide their output and this will result in no tax revenues for the Elite as in Acemoglu (2008). The Elite announces a sequence of tax levels at period 0, before agents make any decisions and it sets the actual tax rate every period t after the agents make their occupational, labor input and investment decisions based on the anticipated policies. The Elite does not have the ability to produce and they receive income in the form of tax revenues and fixed external inflows (foreign aid F and external debt D). I assume that every period the Elite is able to issue a fixed amount of foreign bonds to raise external debt, which is paid with interest by the rest of the agents in the following period. Thus, in any period t the economy holds a current account deficit as is the case for most of the developing small open economies.

The Elite pays patronage to maintain the connections and to remain in power. The Elite can be overthrown with some probability $\delta(\tau_t^c, \tau_t^n)$, which depends on the tax rate τ_t^c

and τ_t^n , for the connected and non-connected entrepreneurs.[4] Besides the preferential tax rates, every period the Elite also pays a fixed lump-sum transfer $P^c > 0$ for each connected agent. The entry barriers per period B^j are fixed and are different for the connected and the non-connected entrepreneurs. For simplicity I consider P^c and B^n as exogenously given and constant overtime.

The expected utility of the Elite at time 0 is given by:

$$\tilde{U_0} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (1 - \delta(\tau_t^c, \tau_t^n))\tilde{c}_t \tag{6}$$

where $\tilde{c}_t$ denotes the consumption of the Elite in period t.

I now make the following simplifying assumption on $\delta(\tau_t^c, \tau_t^n)$ the Elite's ability to lose power.

Assumption 2. *Let us assume that the Elite's ability to lose power satisfies:*

$$\delta(\tau_t^c, \tau_t^n) = 1 \quad if \quad \tau_t^n < \alpha$$

Assumption 2 states that if the taxes on the non-connected are lower than the maximum possible tax rate $\tau_t^n < \alpha$, then there will be no support from the connected individuals towards the Elite to remain in power. This also implies that the tax rate on the connected entrepreneurs in any period will always be lower than or equal to that of the non-connected entrepreneurs, $\tau_t^c \leq \tau_t^n = \alpha$. Therefore, in equilibrium the Elite will always choose $\tau_t^n = \alpha$, so I will abuse the notation and write $\delta(\tau_t^c, \tau_t^n)$ as $\delta(\tau_t^c)$ and assume $\tau_t^n = \alpha$, where $\delta(\tau_t^c)$ is an increasing function of τ_t^c and $\delta'(\tau_t^c) > 0$.

Market Clearing Labor market clearing requires the total demand for labour to be equal to the supply. Given that entrepreneurs also work as production workers in their own firm, the total supply of labour is equal to one at any point in time.

Let N_{Vt}^m be the measure of entrepreneurs of type m in equilibrium in period t.

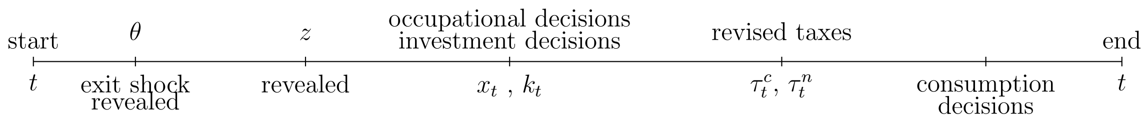
The market clearing condition is given by:

$$\bar{L} \cdot \sum_{\substack{z\in\{H,L\} \\ s\in\{e,r\}}} (N_{Vt}^{nzs} + N_{Vt}^{czs}) = 1. \tag{7}$$

Which implies that in period t in equilibrium, the measure of entrepreneurs is $\frac{1}{\bar{L}}$.

[4] Notice from the assumptions made in the model it will never be the case that the Elite will be out of power or select the level of tax rates where $\delta(\tau_t^c, \tau_t^n) = 1$. However, for clarity we assume that if the Elite ever is removed from power it will be replaced with an identical Elite with same preferences

Timing of Events The timing of the events in this economy is as follows: At the start of the economy at period 0 the Elite announces a sequence of policies p^t. At the beginning of each period t the existing entrepreneurs face the exogenous exit shock and a fraction θ of them lose their existing entrepreneurship status. Each agent's entrepreneurial skill z is realized. All agents make their individual occupational choices x_t, entrepreneurs make investment decisions k_t contingent on the announced sequence of taxes. The Elite decides whether to revise the announced period tax rates τ_t^c and $\tau_t^n = \alpha$ on entrepreneurs. The consumption decisions are made.



4.2 Equilibrium

I divide the definition of the equilibrium in two parts: one involving the agents' choices, given the announced policy sequence (economic equilibrium) and the other involving policy choice by the Elite, taking into account how its policy choices affect agents' choices (political equilibrium).

Economic Equilibrium

Given the timing of the events the agents make the occupational decisions based on the sequence of announced policies p^t by the Elite. The economic equilibrium solves the agents problem based on the announced policies p^t. In equilibrium given consistency in beliefs the announced taxes are equal to the actual optimal taxes set by the Elite.

Profit Maximization Let us define $b^j = B^j/\bar{L}$ as the per worker entry cost, where $b^c = 0$ for the connected agents.

Given wages w_t and expected policies τ_t^c, τ_t^n and given the fact that $\ell_t = \bar{L}$ the profit return gross of the cost of entry barrier of an entrepreneur of type jz is then given by:

$$\pi_t^{jz}(k_t^{jz}, \tau_t^j, w_t) = \frac{1-\tau_t^j}{1-\alpha}(A^z)^{\alpha}(k_t^{jz})^{1-\alpha}(\bar{L})^{\alpha} - w_t\bar{L} - k_t^{jz} \tag{8}$$

The net gain to an entrepreneur who has to pay the entry cost $b^j\bar{L}$ is then given by:

$$\pi_t^{jz}(k_t^{jz}, \tau_t^j, w_t) - b^j\bar{L}$$

Note that since we have normalized the fixed entry cost for the agents with connections to zero, their net gain is always equal to their gross profits.

The profit maximizing optimal investment by an entrepreneur of type jz is then given by:

$$(k_t^{jz})^* = (1-\tau_t^j)^{1/\alpha} A^z \bar{L} \tag{9}$$

Notice that the optimal investment is higher for $z = H$ and decreasing in the tax rate and does not depend on the ownership status of an agent s.

Let K_t be the aggregate capital invested by all entrepreneurs at time t. This is given by:

$$K_t = \sum_{z\in\{H,L\}} (k_t^{nz} N_{Vt}^{nz} + k_t^{cz} N_{Vt}^{cz}) \tag{10}$$

Replacing (9) into (8) we obtain the equilibrium optimal profit gross of the cost of the entry barriers is then given by:

$$\pi_t^{jz}(\tau_t^j, w_t) = \frac{\alpha}{1-\alpha}(1-\tau_t^j)^{1/\alpha} A^z \bar{L} - w_t \bar{L} \tag{11}$$

Occupational Decisions Agents choose the occupation that maximizes their expected lifetime utility at time t. Given the sequence of policies and wages $q^t = (p^t, w^t)$, I define V_t^m as the value function of becoming an entrepreneur at time t gross of entry costs and W_t^m as the value function of being a worker at time t. The expected lifetime utility of the agent at time t is equal to:

$$U_t^{jzs} = \max\{V_t^{jz} - \mathbb{I}_{\{s=r\}} b^j \bar{L}, W_t^{jz}\} \tag{12}$$

where the indicator variable $\mathbb{I}_{\{s=r\}} = 1$, if the agent is a new entrepreneur and $\mathbb{I}_{\{s=r\}} = 0$, if the agent is an existing entrepreneur as only the new entrepreneurs pay the entry barrier.

The value function of being an entrepreneur at time t for type jz gross of entry costs is:

$$V_t^{jz} = w_t + P^j + \pi_t^{jz}(\tau_t^j, w_t) - D_{t-1}(1+r^*) + \beta((1-\theta)(CV_{t+1}^{jz}) + \theta(CW_{t+1}^{jz})) \tag{13}$$

The expression (13) shows that the value of being an entrepreneur at time t depends on their consumption at time t, which is equal to the net of wages earned w_t, patronage received P^j and net profits from the firm $\pi_t^{jz}(\tau_t^j)$, after deducting the repayment of the previous period external debt $D_{t-1}(1+r^*)$ and on the discounted continuation value of an entrepreneur CV_{t+1}^{jz} and a worker CW_{t+1}^{jz}, with probabilities $1-\theta$ and θ, respectively.

CV_{t+1}^{jz} is the continuation value of an entrepreneur of type jz at period t entering period

$t+1$ as an entrepreneur ($s_{t+1}=e$), and is given by :

$$CV_{t+1}^{jz} = \sigma^z \max\{W_{t+1}^{jH}; V_{t+1}^{jH}\} + (1-\sigma^z)\max\{W_{t+1}^{jL}; V_{t+1}^{jL}\} \tag{14}$$

The expression (14) says that with probability σ^z in the next period the agent is high skilled and chooses the maximum between remaining an entrepreneur or becoming a worker, and with probability $1-\sigma^z$ in the next period the agent is low skilled and chooses the maximum between remaining an entrepreneur or becoming a worker.

CW_{t+1}^{jz} is the continuation value of a worker of type jz at period t entering period $t+1$ as a worker ($s_{t+1}=r$), and is given by:

$$CW_{t+1}^{jz} = \sigma^z \max\{W_{t+1}^{jH}; V_{t+1}^{jH} - b^j\bar{L}\} + (1-\sigma^z)\max\{W_{t+1}^{jL}; V_{t+1}^{jL} - b^j\bar{L}\} \tag{15}$$

The expression (15) says that with probability σ^z in the next period the agent is high skilled and chooses the maximum between becoming an entrepreneur or remaining a worker, and with probability $1-\sigma^z$ in the next period the agent is low skilled and chooses the maximum between becoming an entrepreneur or remaining a worker. Notice that as $b^c=0$ for an agent type cz at time t, $CV_{t+1}^{cz} = CW_{t+1}^{cz}$.

Similarly, the value function of becoming a worker at time t for type jz is :

$$W_t^{jz} = w_t + P^j - D_{t-1}(1+r^*) + \beta CW_{t+1}^{jz} \tag{16}$$

The expression (16) shows that the value of being a worker at time t depends on their consumption at time t, which is equal to the net of wages earned w_t and the patronage received P^j, after deducting the repayment of the previous period external debt $D_{t-1}(1+r^*)$ and on the discounted continuation value of remaining a worker CW_{t+1}^{jz}.

Definition of economic equilibrium Given the sequence of tax policies $p^* = \{p_t\}_{t=0}^{\infty}$, an economic equilibrium is defined as a sequence of agents decisions $x^* = \{x_t\}_{t=0}^{\infty}$ and $k^* = \{k_t\}_{t=0}^{\infty}$, a sequence of wages $w^* = \{w_t\}_{t=0}^{\infty}$, and entrepreneurship measure $N_V^{m*} = \{N_{Vt}^*\}_{t=0}^{\infty}$ s.t:

- Given $p^*$and w^* and an agent's decisions, (x^*, k^*) solves the problem of the agents given by (12)

- All markets clear

- N_V^{m*} is consistent with the agents' individual decisions at any period t.

Solving For The Economic Equilibrium

Net value gain of entrepreneurship Given the announced set of policies p^t, the occupational choice of agents at time t depends on their net value gain of becoming an entrepreneur at time t (NG). This is equal to the difference between the value of an entrepreneur and a worker, after deducting the entry costs for non-existing entrepreneurs at time t. I derive the net value gain of entrepreneurship for an agent at time t conditional on their type as the following:

$$NG_t^m = V_t^m - W_t^m - \mathbb{I}_{\{s_t=r\}} b^j \bar{L} \tag{17}$$

Using (13), (16) and (17) the net gain of an entrepreneur of type $m = jzs$ at time t is given by the following expression:

$$NG_t^m = \frac{\alpha}{1-\alpha}(1-\tau_t^j)^{1/\alpha} A^z \bar{L} - w_t \bar{L} + \beta(1-\theta)(CV_{t+1}^{jz} - CW_{t+1}^{jz}) - \mathbb{I}_{\{s=r\}} b^j \bar{L} \tag{18}$$

Thus given the above expression for the net gain of entrepreneurship, an agent type m strictly prefers to become an entrepreneur at time t, if their $NG_t^m > 0$ and be indifferent between becoming an entrepreneur or a worker if their $NG_t^m = 0$ and becomes a worker if $NG_t^m < 0$. If an individual is an existing entrepreneur their net value gain at time t will just depend on the difference in the value functions of remaining an entrepreneur and becoming a worker. However, if an agent is a non-owner of a firm then their net value gain of entrepreneurship will depend on the net difference between the value of becoming an entrepreneur and remaining a worker after paying the entry costs.

Given assumption 2, in what follows I set $\tau_t^n = \alpha$.

Wage Thresholds and Equilibrium wages Notice from the net gain expression (18), the occupational decision and, therefore, the aggregate labour demand is a function of the period wage. In what follows, I determine the equilibrium wages by deriving the aggregate labour demand and supply functions in the model. The aggregate supply function is constant at 1. The aggregate demand function depends on the measure of agents of each type that become entrepreneurs. To derive the labor demand function generated by each agent type m in equilibrium at time t, the first step is to solve for the wage thresholds that make a given agent of type m indifferent between becoming an entrepreneur or staying a worker.

The value for the wage threshold for an agent type m at time t 'w_t^m' are derived by setting $NG_t^m = 0$. The individual becomes an entrepreneur if the equilibrium wage $w_t < w_t^m$, is indifferent if $w_t = w_t^m$ and becomes a worker if the equilibrium wage is above threshold. The threshold wages are as follows:

For a connected agent with skill z the threshold wage is:

$$w_t^{cz} = \frac{\alpha}{1-\alpha}(1-\tau_t^c)^{1/\alpha}A^z \tag{19}$$

For a non-connected worker with skill z:

$$w_t^{nzr} = \frac{\alpha}{1-\alpha}[(1-\alpha))^{1/\alpha}A^z] - b^n + \frac{\beta(1-\theta)(CV_{t+1}^{nz} - CW_{t+1}^{nz})}{\bar{L}} \tag{20}$$

For a non-connected entrepreneur with skill z:

$$w_t^{nze} = \frac{\alpha}{1-\alpha}(1-\alpha))^{1/\alpha}A^z + \frac{\beta(1-\theta)(CV_{t+1}^{nz} - CW_{t+1}^{nz})}{\bar{L}} \tag{21}$$

In what follows I derive relationships among these thresholds that will be useful in determining the aggregate labor demand curve.

Lemma 4.4. *The threshold wages satisfy the following inequalities:*

$$\begin{aligned} w_t^{cH} &> w_t^{cL} \\ w_t^{nHe} > w_t^{nHr} &> w_t^{nLr} \\ w_t^{nLe} &> w_t^{nLr} \end{aligned} \tag{22}$$

Proof: Appendix A.3.

The ordering between w_t^{cL}, w_t^{nHr}, w_t^{nLe} depends on the policy sequence p^t. The next lemmas derive conditions that determine how these threshold wages are ordered.

Lemma 4.5. *Given* p^t *:*

(i) $w_t^{cL} \geq w_t^{nHe}$ *if and only if* $(1-\alpha))^{1/\alpha}A^H + \frac{\beta(1-\theta)(CV_{t+1}^{nH} - CW_{t+1}^{nH})}{\bar{L}} \leq (1-\tau_t^c)^{1/\alpha}A^L$

(ii) $w_t^{cL} > w_t^{nHr}$ *if and only if* $\frac{\alpha}{1-\alpha}[(1-\tau_t^c)^{1/\alpha}A^L - ((1-\alpha))^{1/\alpha}A^H + \frac{\beta(1-\theta)(CV_{t+1}^{nH} - CW_{t+1}^{nH})}{\bar{L}})] > -b^n$

(iii) $w_t^{nHr} > w_t^{nLe}$ *if and only if* $\frac{\alpha}{1-\alpha}(1-\alpha))^{1/\alpha}(A^H - A^L) + \frac{\beta(1-\theta)((CV_{t+1}^{nH} - CW_{t+1}^{nH}) - (CV_{t+1}^{nL} - CW_{t+1}^{nL}))}{\bar{L}} > b^n$

Proof. Follows directly from the definitions of the wage thresholds. □

Based on Lemmas 4.4 to 4.5 five types of potential complete orderings of the wage thresholds are possible:

$$\textbf{Case 1:} \qquad w_t^{cH} > \boldsymbol{w_t^{cL}} \geq \boldsymbol{w_t^{nHe}} > \boldsymbol{w_t^{nHr}} \geq \boldsymbol{w_t^{nLe}} > w_t^{nLr} \tag{23}$$

when p^t satisfy:

$$\begin{aligned} &\frac{\alpha}{1-\alpha}[(1-\tau_t^c)^{1/\alpha}A^L - (1-\alpha))^{1/\alpha}A^H - \frac{\beta(1-\theta)(CV_{t+1}^{nH} - CW_{t+1}^{nH})}{\bar{L}}] \geq 0 > -b^n \\ \& \quad &\frac{\alpha}{1-\alpha}(1-\alpha)^{1/\alpha}(A^H - A^L) + \frac{\beta(1-\theta)((CV_{t+1}^{nH} - CW_{t+1}^{nH}) - (CV_{t+1}^{nL} - CW_{t+1}^{nL}))}{\bar{L}} > b^n \end{aligned}$$

$$\textbf{Case 2:} \qquad w_t^{cH} > \boldsymbol{w_t^{nHe}} \geq \boldsymbol{w_t^{cL}} > \boldsymbol{w_t^{nHr}} \geq \boldsymbol{w_t^{nLe}} > w_t^{nLr} \tag{24}$$

when p^t satisfy:

$$\begin{aligned} &0 \geq \frac{\alpha}{1-\alpha}[(1-\tau_t^c)^{1/\alpha}A^L - (1-\alpha))^{1/\alpha}A^H - \frac{\beta(1-\theta)(CV_{t+1}^{nH} - CW_{t+1}^{nH})}{\bar{L}}] > -b^n \\ \& \quad &\frac{\alpha}{1-\alpha}(1-\alpha))^{1/\alpha}(A^H - A^L) + \frac{\beta(1-\theta)((CV_{t+1}^{nH} - CW_{t+1}^{nH}) - (CV_{t+1}^{nL} - CW_{t+1}^{nL}))}{\bar{L}} > b^n \end{aligned}$$

$$\textbf{Case 3:} \qquad w_t^{cH} \geq w_t^{nHe} > \boldsymbol{w_t^{nHr}} \geq \boldsymbol{w_t^{cL}} \geq w_t^{nLe} > w_t^{nLr} \tag{25}$$

when p^t satisfy:

$$\frac{\alpha}{1-\alpha}[(1-\tau_t^c)^{1/\alpha}A^L - (1-\alpha))^{1/\alpha}A^H - \frac{\beta(1-\theta)(CV_{t+1}^{nH} - CW_{t+1}^{nH})}{\bar{L}}] \leq -b^n$$

$$\textbf{Case 4:} \qquad w_t^{cH} > \boldsymbol{w_t^{cL}} > \boldsymbol{w_t^{nHe}} > \boldsymbol{w_t^{nLe}} \geq \boldsymbol{w_t^{nHr}} \geq w_t^{nLr} \tag{26}$$

when p^t satisfy:

$$\begin{aligned} &\frac{\alpha}{1-\alpha}[(1-\tau_t^c)^{1/\alpha}A^L - (1-\alpha))^{1/\alpha}A^H - \frac{\beta(1-\theta)(CV_{t+1}^{nH} - CW_{t+1}^{nH})}{\bar{L}}] \geq 0 > -b^n \\ \& \quad &\frac{\alpha}{1-\alpha}(1-\alpha))^{1/\alpha}(A^H - A^L) + \frac{\beta(1-\theta)((CV_{t+1}^{nH} - CW_{t+1}^{nH}) - (CV_{t+1}^{nL} - CW_{t+1}^{nL}))}{\bar{L}} \leq b^n \end{aligned}$$

Case 5: $$w_t^{cH} > \boldsymbol{w_t^{nHe} > w_t^{cL} > w_t^{nLe} \geq w_t^{nHr}} \geq w_t^{nLr} \tag{27}$$

when p^t satisfy:

$$0 \geq \frac{\alpha}{1-\alpha}[(1-\tau_t^c)^{1/\alpha}A^L - (1-\alpha))^{1/\alpha}A^H - \frac{\beta(1-\theta)(CV_{t+1}^{nH} - CW_{t+1}^{nH})}{\bar{L}}] > -b^n$$

$$\& \quad \frac{\alpha}{1-\alpha}(1-\alpha))^{1/\alpha}(A^H - A^L) + \frac{\beta(1-\theta)((CV_{t+1}^{nH} - CW_{t+1}^{nH}) - (CV_{t+1}^{nL} - CW_{t+1}^{nL}))}{\bar{L}} \leq b^n$$

To simplify the number of cases to consider, we introduce an additional assumption on parameters, which is satisfied in our calibration, that ensures that $V_t^{nL} - W_t^{nL} < 0 \; \forall \; t$, so that $w_t^{nHr} > w_t^{nLe}$ and eliminates cases 4 and 5. Appendix A.4 proves this formally.

Assumption 3. *The entry barrier per worker for the non-connected satisfy the following:*

$$\frac{\frac{\alpha}{1-\alpha}(1-\alpha)^{1/\alpha}(A^H - A^L)}{1-\beta(1-\theta)\sigma^H} > b^n.$$

This assumption guarantees that $w_t^{nHr} > w_t^{nLe}$ for all t, effectively eliminating cases 4 and 5. It achieves this by ensuring that entry barriers are not excessively high. High entry barriers diminish the net value gain of entrepreneurship for the non-connected high skilled non entrepreneurs. This, in turn, allows low-skilled non-connected existing entrepreneurs to either remain or transition into entrepreneurship.

Figures 3 and 4 display the labour demand functions for selected cases 2 and 3.[5] Notice that labour demand is a decreasing step function of the wage rate. Each step shows the measure of total labour demanded by entrepreneurs of type m at time t, and the length of the step depends on the measure of agents of type m at time t. The equilibrium wage is at the point where labour demand is equal to 1, which, in turn, depends on assumptions on the initial distribution of agents.

[5] Note that case 1 has a labour demand function similar to figure 3 for case 2 with $w_t^{cL} > w_t^{nHe}$.

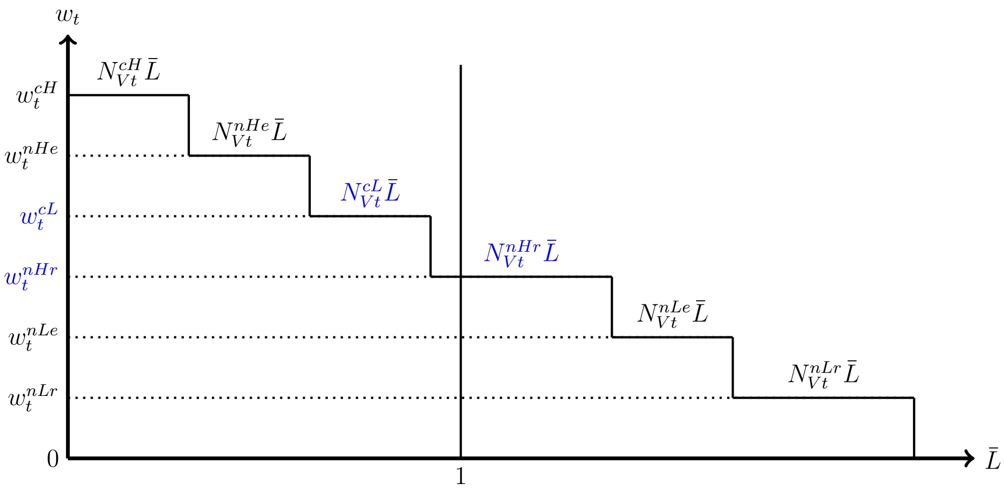


Figure 3: Labor demand in case 2

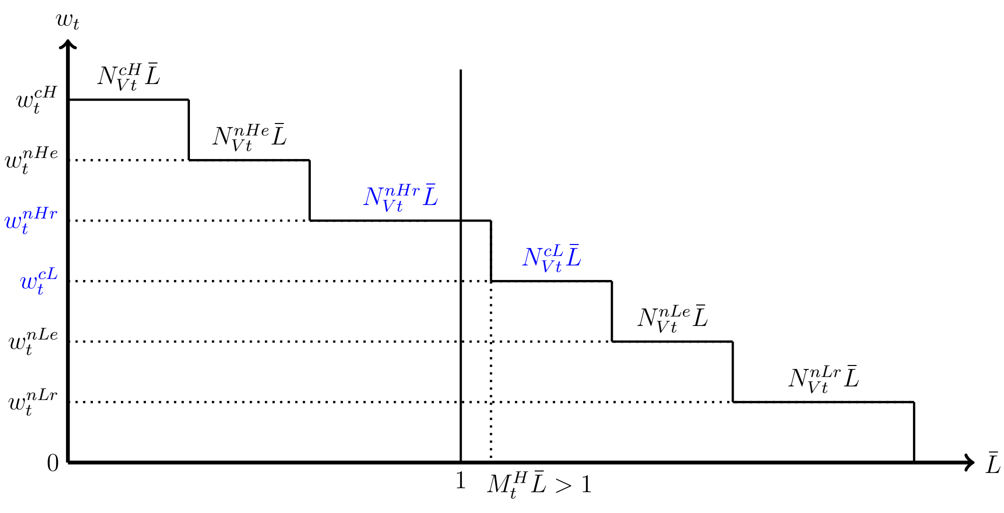


Figure 4: Labor demand in case 3

Lemma 4.6. *Under assumption 1, the equilibrium wage is $w_t^* = w_t^{nHr}$ for any policy sequence p^t.*

Proof: Appendix A.3.

In what follows, I characterize the economic equilibrium as a function of sequence $\{\tau_t^c\}_{n=t}^{\infty}$ and exogenous parameters.

Lemma 4.7. *Under lemma 4.6, assumption 3 and given that $\tau_t^n = \alpha \; \forall t$,*

$$\frac{\beta(CV_{t+1}^{nH} - CW_{t+1}^{nH})}{\bar{L}} = \beta\sigma^H b^n.$$

Proof: Appendix A.3.

Notice from figures 3 and 4 that the labor supply is drawn as a constant at 1 and depicts the equilibrium wage $w_t^* = w_t^{nHr}$ for the cases 2 and 3, where the total labor demanded is equal to the labor supply.

In what follows I characterize the economic equilibrium as a function of tax policy sequence p^t and exogenous parameters.

Characterization of Economic Equilibrium

I now define a threshold for the taxes on the connected entrepreneurs each period, in terms of the exogenous parameters of the model, which will help characterize the economic equilibrium, as follows:

Definition 1. *Let us define $\bar{\tau}$ as the threshold value of τ_t^c for which $w_t^{nHr} = w_t^{cL}$ as given by:*

$$\bar{\tau}^c = \left(\frac{b^n(1-\beta(1-\theta)\sigma^H)^{\frac{(1-\alpha)}{\alpha}} - (1-\alpha)^{\frac{1}{\alpha}} A^H}{A^L}\right)^{\alpha} + 1.$$

Proposition 1. *Under assumptions 1 and 3, given the policy sequence p^t there exists a unique economic equilibrium. In equilibrium $w_t^* = w_t^{nHr}$ for all t and the measures of some of the entrepreneurs of each type at any time t take the form:*

(a) $(N_{Vt}^{cH})^* = \phi^c M_t^H$

(b) $(N_{Vt}^{nL})^* = 0$

While the measures of the remaining entrepreneur types takes one of the two forms, based on the policy τ_t^c:

1. *If* $\tau_t^c < \bar{\tau}$ *, then:*

   (c) $(N_{Vt}^{cL})^* = \phi^c - N_{Vt}^{cH}$

   (d) $(N_{Vt}^{nHe})^* = (1-\theta)\sigma^H N_{Vt-1}^{nH}$

   (e) $(N_{Vt}^{nHr})^* = (\frac{1}{L} - \phi^c) - N_{Vt}^{nHe}$

So that all connected are entrepreneurs. We call this situation a ***type 1 equilibrium****.*
OR,

2. *If* $\tau_t^c \geq \bar{\tau}$*, then:*

   (c) $(N_{Vt}^{cL})^* = 0$

   (d) $(N_{Vt}^{nHe})^* = (1-\theta)\sigma^H (N_{Vt-1}^{nH})$

   (e) $(N_{Vt}^{nHr})^* = (\frac{1}{L} - N_{Vt}^{cH}) - N_{Vt}^{nHe}$

So that only high skilled connected and non-connected are entrepreneurs. We call this situation a ***type 2 equilibrium****.*

Proof: Appendix A.3.

This proposition establishes that, if τ_t^c satisfies $\tau_t^c < \bar{\tau}$ (type 1 equilibrium), then in equilibrium all connected (high and low skilled) agents are entrepreneurs, this creates a misallocation of resources, since low skilled entrepreneurs produce when high skilled are kept out of the market. The favourable tax rates for the connected entrepreneurs makes it more attractive and profitable for the low productivity connected entrepreneurs to enter the market, crowding out entrepreneurship for highly productive non-connected.

If τ_t^c satisfies $\tau_t^c \geq \bar{\tau}$ (type 2 equilibrium), then in equilibrium all connected high skilled and a few non-connected high skilled individuals are entrepreneurs. Therefore, only high skilled agents remain or become entrepreneurs and there is no resource misallocation.

Notice that I have assumed that if $\tau_t^c = \bar{\tau}$, then high skilled agents enter first and the economy is in type 2 equilibrium. Also notice that the total number of non-connected high skilled entrepreneurs is higher in type 2, since $N_{Vt}^{cL} = 0$.

Aggregate Resource Constraint

For simplicity let C_t denote the aggregate consumption of all connected and non-connected agents of different types, and K_t denote the capital invested by all the non-connected and connected entrepreneurs at the end of period t. The aggregate resource constraint of the economy is then given by:

$$Y_t + D_t + F = \tilde{c}_t + C_t + K_t + B^n N_{Vt}^{nHr} + D_{t-1}(1 + r^*) \tag{28}$$

where at the end of each period t, Y_t is the total output in the economy produced by the connected and non-connected agents, $B^n N_{Vt}^{nHr}$ is the total expenses on the entry barriers by the new entrepreneurs, N_{Vt}^{nHr} is the measure of non-connected new entrepreneurs in equilibrium, D_t is the new external debt and $D_{t-1}(1 + r^*)$ is the total debt repayment of the previous period debt plus the interest owed on D_{t-1} at the world interest rate r^*.

Characterization of Stationary Economic Equilibrium

Using the threshold for the taxes on the connected entrepreneurs $\bar{\tau}$ and the exogenous parameters of the model, I characterize the stationary economic equilibrium, as follow:

Proposition 2. *Assume that $\tau_t^c = \tau^c$ for all t and assume that assumptions 1 and 3 holds, there exists a unique stationary economic equilibrium. In equilibrium $w_{t+1}^* = w_t^* = w^{nHr}$ for all t and the measures of some of the entrepreneurs of each type take the form:*

(a) $(N_V^{cH})^* = \phi^c M^H$

(b) $(N_V^{nL})^* = 0$

1. *If $\tau^c < \bar{\tau}$, then:*

   (c) $(N_V^{cL})^* = \phi^c(1 - M^H)$

   (d) $(N_V^{nHe})^* = (1 - \theta)\sigma^H(\frac{1}{L} - \phi^c)$

   (e) $(N_V^{nHr})^* = (1 - (1 - \theta)\sigma^H)(\frac{1}{L} - \phi^c)$

*So that all connected are entrepreneurs. We call this situation a **type 1 stationary equilibrium**.*

OR,

2. *If $\tau^c \geq \bar{\tau}$, then:*

   (c) $(N_V^{cL})^* = 0$

   (d) $(N_V^{nHe})^* = (1 - \theta)\sigma^H(\frac{1}{L} - \phi^c M^H)$

(e) $(N_V^{nHr})^* = (1-(1-\theta)\sigma^H)(\frac{1}{L} - \phi^c M^H)$

So that only high skilled connected and non-connected are entrepreneurs. We call this situation a ***type 2 stationary equilibrium****.*

Proof: Appendix A.3.

This proposition establishes that, if τ^c satisfies $\tau^c < \bar{\tau}$ (type 1 stationary equilibrium), then in a stationary equilibrium all connected (high and low skilled) agents are entrepreneurs, this creates a misallocation of resources, since low skilled entrepreneurs produce when high skilled are kept out of the market. The favourable tax rates for the connected entrepreneurs makes it more attractive and profitable for the low productivity connected entrepreneurs to enter the market, crowding out entrepreneurship for highly productive non-connected.

If τ^c satisfies $\tau^c \geq \bar{\tau}$ (type 2 stationary equilibrium), then in a stationary equilibrium all connected high skilled and a few non-connected high skilled individuals are entrepreneurs. Therefore, only high skilled agents remain or become entrepreneurs and there is no resource misallocation.

Notice that I have assumed that if $\tau^c = \bar{\tau}$, then high skilled agents enter first and the economy is in a type 2 stationary equilibrium. Also notice that the total number of non-connected high skilled entrepreneurs and the non-connected high skilled existing entrepreneurs is higher in type 2, since $N_V^{cL} = 0$. This is important, as it shows that the measure of existing non-connected entrepreneurs making positive profits are higher under a type 2 stationary equilibrium and thus, the total welfare for non-connected agents will be higher under a type 2 stationary equilibrium.

Political Equilibrium

Elite's problem Given the sequence of announced policies $\hat{p^t}$ each period the Elite chooses the actual tax rate sequence $\{\tau_t^c\}_{n=t}^{\infty}$ to maximize their preferences taking into account that their choice of taxes affect the occupational decisions of the agents determined in the economic equilibrium. Using the notation discussed in subsection 4.1, the Elite's lifetime problem is as follows:

$$\max_{\{\tau_t^c\}_{n=t}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (1-\delta(\tau_t^c))\tilde{c}_t$$

Subject to:

$$T_t^n + T_t^c + F + D - P^c\phi^c \geq \tilde{c}_t \ \ \forall t$$

$$\tilde{c}_t \geq 0 \ \ \forall t$$

$$\tau_t^n = \alpha \ \ \forall t$$

Where:

1. $T_t^c = \frac{1}{1-\alpha}\tau_t^c(1-\tau_t^c)^{\frac{1-\alpha}{\alpha}}\bar{L}[A^H N_{Vt}^{cH} + A^L N_{Vt}^{cL}]$is the total tax revenue from the connected entrepreneurs.

2. $T_t^n = \frac{1}{1-\alpha}\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}\bar{L}A^H[\frac{1}{\bar{L}} - (N_{Vt}^{cH} + N_{Vt}^{cL}]$ is the total tax revenue from the non-connected entrepreneurs.

Notice from the Elite's budget constraint that it needs to raise at least $P^c\phi^c$ for their consumption to be positive. Henceforth for simplicity, I assume that $P^c\phi^c$ is small compared to the total revenues in the form of taxes and the external flows for the Elite. Therefore, I only consider cases where the constraint on the consumption $\tilde{c}_t \geq 0$ is non-binding for all t.

Also notice that the period level of tax revenues from the non-connected entrepreneurs does not depend on their skill type, it only depends on the total measure of non-connected high skilled entrepreneurs. Furthermore, due to the two types of economic equilibria, $N_{Vt}^{cL}(\tau_t^c)$ have a kink at $\tau_t^c = \bar{\tau}$, the problem is not differentiable and there is a discontinuity on the agent's decisions when $\tau_t^c = \bar{\tau}$.

The next lemma shows that tax revenues at period t depend only on the period tax rate τ_t^c and the (exogenously determined) measure of high skilled entrepreneurs M_t^H, and not on previous period decisions. This simplifies enormously the task of computing the political equilibrium.

Lemma 4.8. *Assume assumption 1 holds. Given M_0^H, it is the case that T_t^c and T_t^n at time t are functions of τ_t^c and M_t^H only.*

Proof: Appendix A.3.

Thus, given lemma 4.8 the Elite's budget constraint in period t depends only on period t variables. Therefore, the problem of the Elite can be reduced to solving an infinite set of static problems, one per period. Thus, I separate the problem of a given period t in two parts: solve for $\tilde{u}_t{}^1$ and $\tilde{u}_t{}^2$, where $\tilde{u}_t{}^1$ is the maximum period utility of the Elite from being in type 1 economic equilibrium, assuming the Elite chooses $\tau_t^c < \bar{\tau}$ and $\tilde{u}_t{}^2$ is the maximum period utility of the Elite from being in type 2 economic equilibrium, assuming that the Elite chooses $\tau_t^c \geq \bar{\tau}$. The Elite then chooses $\tilde{u}_t$, which is the best of these two options in each period. The Elite's lifetime utility function can then be expressed as:

$$\tilde{U}_t = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \max\{\tilde{u}_t{}^1, \tilde{u}_t{}^2\} \tag{29}$$

Where:

$$\tilde{u}^1 = \max_{\tau_t^c}(1 - \delta(\tau_t^c))\tilde{c}_t^{\,1} \tag{30}$$

Subject to:

$$(T_t^c)^1(\tau_t^c) + (T_t^n)^1(\tau_t^c) + F + D - P^c\phi^c \geq \tilde{c}_t^{\,1}$$

$$\tau_t^n = \alpha$$

$$\tau_t^c \leq \bar{\tau}$$

and

$$\tilde{u}_t^{\,2} = \max_{\tau_t^c}(1 - \delta(\tau_t^c))\tilde{c}_t^{\,2} \tag{31}$$

Subject to:

$$(T_t^c)^2(\tau_t^c) + (T_t^n)^2(\tau_t^c) + F + D - P^c\phi^c \geq \tilde{c}_t^{\,2}$$

$$\tau_t^n = \alpha$$

$$\tau_t^c \geq \bar{\tau}$$

Notice that for the purpose of solving the model and to guarantee a solution $\tilde{u}_t^{\,1}$, we use the weak inequality $\tau_t^c \leq \bar{\tau}$.

In what follows, I solve for the values $(\tilde{\tau}_t^c)^1$ and $(\tilde{\tau}_t^c)^2$, that maximize $\tilde{u}_t^{\,1}$ and $\tilde{u}_t^{\,2}$ without putting the additional constraint on τ_t^c. Next, I verify whether each solution satisfies the corresponding constraint on τ_t^c. If it is then the solution to the constrained problem (30) and (31) is given by $(\tau_t^c)^i = (\tilde{\tau}_t^c)^i$. If it is not then $(\tau_t^c)^i = \bar{\tau}$. I then discuss potential cases where $\tilde{u}_t^{\,2}$ might be greater or lower than $\tilde{u}_t^{\,1}$. I present this solution approach in more detail below.

Type 1 Economic Equilibrium (determining $\tilde{u}_t^{\,1}$) The solution to the unconstrained problem (30) assuming type 1 equilibrium is:

$$\tilde{u}_t^{\,1} = \max(1 - \delta(\tau_t^c))\tilde{c}_t^{\,1} \tag{32}$$

Subject to:

$$(T_t^c)^1(\tau_t^c) + (T_t^n)^1(\tau_t^c) + F + D - P^c\phi^c \geq (\tilde{c}_t)^1$$

$$\tau_t^c \leq \bar{\tau}$$

The First order condition is given by:

$$[(1 - \delta(\tau_t^c))[(T_t^c)^1]'(\tau_t^c) - \delta'(\tau_t^c)(TR_t)^1(\tau_t^c)] = 0 \tag{33}$$

Where $(TR_t)^1(\tau_t^c) = (T_t^c)^1(\tau_t^c) + (T_t^n)^1(\tau_t^c) + F + D - P^c\phi^c$,
$[(T_t^c)^1]'(\tau_t^c) = \bar{L}(\phi^c M_t^H A^H + \phi^c(1 - M_t^H)A^L)(1 - \tau_t^c)^{\frac{1-\alpha}{\alpha}}(\frac{\alpha - \tau_t^c}{\alpha(1-\alpha)(1-\tau_t^c)})$ and $\delta'(\tau_t^c) > 0$.

Type 2 Economic Equilibrium (determining $\tilde{u_t}^2$) The solution to the unconstrained problem (31) assuming type 2 equilibrium is:

$$\tilde{u_t}^2 = \max(1 - \delta(\tau_t^c))\tilde{c_t}^2 \tag{34}$$

Subject to:

$$(T_t^c)^2(\tau_t^c) + (T_t^n)^2(\tau_t^c) + F + D - P^c\phi^c \geq (\tilde{c}_t)^2$$

$$\tau_t^c \geq \bar{\tau}$$

The First order condition is given by:

$$[(1 - \delta(\tau_t^c))[(T_t^c)^2]'(\tau_t^c) - \delta'(\tau_t^c)(TR_t)^2(\tau_t^c)] = 0 \tag{35}$$

Where $(TR_t)^2(\tau_t^c) = (T_t^c)^2(\tau_t^c) + (T_t^n)^2(\tau_t^c) + F + D - P^c\phi^c$,
$[(T_t^c)^2]'(\tau_t^c) = \bar{L}(\phi^c M_t^H A^H)(1 - \tau_t^c)^{\frac{1-\alpha}{\alpha}}(\frac{\alpha - \tau_t^c}{\alpha(1-\alpha)(1-\tau_t^c)})$ and $\delta'(\tau_t^c) > 0$.

The following lemmas show how the solutions to the unconstrained problems above, determines the solution to the Elite's problem (29).

Lemma 4.9. *If $(\tilde{\tau}_t^c)^1 > \bar{\tau}$ and $(\tilde{\tau}_t^c)^2 < \bar{\tau}$ then $(\tau_t^c)^1 = (\tau_t^c)^2 = \bar{\tau}$.*

Proof: Appendix A.3.

Lemma 4.10. *Under assumptions 1 and 2, $(TR_t)^2(\tau_t^c) > (TR_t)^1(\tau_t^c)$.*

Proof: Appendix A.3.

Lemma 4.11. *Assume assumption 1 holds. If $(\tau_t^c)^2 > \bar{\tau}$ then $(\tau_t^c)^1 = \bar{\tau}$.*

Proof: Appendix A.3.

The above lemmas help to narrow down some potential solutions to the Elite's period problem given by (29). In particular, lemma 4.9 show that if the solution $(\tilde{\tau}_t^c)^i$ does not satisfy the constraint on τ_t^c for both type 1 and type 2 equilibrium, then the optimal solution to (30) and (31) will be $\bar{\tau}$. Lemma 4.10, then shows that for any given $(\tau_t^c)^1 = (\tau_t^c)^2$, the total revenues for the Elite will be higher under a type 2 equilibrium, so that $\tilde{u_t}^1 < \tilde{u_t}^2$. Lastly, lemma 4.11 show that if $(\tau_t^c)^2$ is an interior solution, then the unconstrained solution $(\tilde{\tau_t^c})^1$

will not satisfy the constraint on τ_t^c and the optimal solution $(\tau_t^c)^1$ will be a corner solution, that is $\bar{\tau}$.

Given the above lemmas, in what follows, I present and discuss some potential cases and possible solutions to the Elite's per period problem given by (29).

Proposition 3. *Assume assumption 1 holds. Then it is the case that:*

1. *If $(\tau_t^c)^1 = (\tau_t^c)^2 = \bar{\tau}$, then $\tilde{u_t}^2 > \tilde{u_t}^1$ and $\tilde{u}_t = \tilde{u_t}^2$*
2. *If $(\tau_t^c)^1 = \bar{\tau}$ and $(\tau_t^c)^2 > \bar{\tau}$, then $\tilde{u_t}^2 > \tilde{u_t}^1$ and $\tilde{u}_t = \tilde{u_t}^2$*

Proof: Appendix A.3.

A third interesting case arises if $(\tau_t^c)^2 = \bar{\tau}$ and $(\tau_t^c)^1 \leq \bar{\tau}$, then it can that either $\tilde{u_t}^2 > \tilde{u_t}^1$ or $\tilde{u_t}^2 < \tilde{u_t}^1$. If this is the case then, depending on the parameters of the model that determine $\bar{\tau}$, the external flows and the functional form for $\delta(\tau_t^c)$, the Elite's utility under type 1 and type 2 economic equilibrium at time t will be compared and which ever is higher will determine the Elite's period utility maximizing policy $(\tau_t^c)^*$. If $\tilde{u_t}^2 > \tilde{u_t}^1$, then $(\tau_t^c)^*= \bar{\tau}$ and $\tilde{u}_t = \tilde{u_t}^2$, and if $\tilde{u_t}^1 > \tilde{u_t}^2$, then $(\tau_t^c)^* = (\tau_t^c)^1$ and $\tilde{u}_t = \tilde{u_t}^1$.

4.2.1 Stationary Equilibrium

A stationary equilibrium will then consist of an equilibrium where the measures of the entrepreneurs of each type N_V^m do not change over time and the optimal policy $(\tau_{t+1}^c)^* = (\tau_t^c)^* = (\tau^c)^*$ $\forall t$. Notice that because the Elite's problem is a sequence of static problems, the stationary problem can also be solved following the procedure discussed above, using the stationary distribution of entrepreneurs given in proposition 2. The Elite's lifetime utility function in a stationary equilibrium can then be expressed as:

$$\tilde{U} = \frac{1}{(1-\beta)} \max\{\tilde{u}^1, \tilde{u}^2\} \tag{36}$$

Where:

$$\tilde{u}^1 = \max_{\tau^c} (1 - \delta(\tau^c))\tilde{c}^1 \tag{37}$$

Subject to:

$$(T^c)^1(\tau^c) + (T^n)^1(\tau^c) + F + D - P^c\phi^c \geq \tilde{c}^1$$

$$\tau^c \leq \bar{\tau}$$

and

$$\tilde{u}^2 = \max_{\tau^c} (1 - \delta(\tau^c))\tilde{c}^2 \tag{38}$$

Subject to:

$$(T^c)^2(\tau^c) + (T^n)^2(\tau^c) + F + D - P^c\phi^c \geq \tilde{c}^2$$

$$\tau^c \geq \bar{\tau}$$

Where,

1. $(T^c)^1(\tau^c) = \frac{1}{1-\alpha}\tau^c(1-\tau^c)^{\frac{1-\alpha}{\alpha}}\bar{L}[\phi^c M^H A^H + \phi^c(1-M^H)A^L]$
2. $(T^n)^1(\tau^c) = \frac{1}{1-\alpha}\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}\bar{L}(\frac{1}{\bar{L}} - \phi^c)A^H$
3. $(T^c)^2(\tau^c) = \frac{1}{1-\alpha}\tau^c(1-\tau^c)^{\frac{1-\alpha}{\alpha}}\bar{L}[\phi^c M^H A^H]$
4. $(T^n)^2(\tau^c) = \frac{1}{1-\alpha}\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}\bar{L}(\frac{1}{\bar{L}} - \phi^c M^H)A^H$

The following proposition characterize the different types of stationary equilibrium that can exist.

Proposition 4. *Assume assumption 1 holds. Let $(\tau^c)^1$ and $(\tau^c)^2$ be the optimal solutions to the stationary problems. Then it is the case that in steady state:*

1. *If $(\tau^c)^1 = (\tau^c)^2 = \bar{\tau}$, then $\tilde{u}^2 > \tilde{u}^1$ and $\tilde{u} = \tilde{u}^2$*
2. *If $(\tau^c)^1 = \bar{\tau}$ and $(\tau^c)^2 > \bar{\tau}$, then $\tilde{u}^2 > \tilde{u}^1$ and $\tilde{u} = \tilde{u}^2$*

Proof: Appendix A.3.

Similar to the non-stationary per period equilibrium in a stationary setting, a third interesting case arises if $(\tau^c)^2 = \bar{\tau}$ and $(\tau^c)^1 \leq \bar{\tau}$, then it can that either $\tilde{u}^2 > \tilde{u}^1$ or $\tilde{u}^2 < \tilde{u}^1$. If this is the case then, depending on the parameters of the model that determine $\bar{\tau}$, the external flows and the functional form for $\delta(\tau^c)$, the Elite's utility under type 1 and type 2 stationary economic equilibrium will be compared and which ever is higher will determine the Elite's steady state period utility maximizing policy $(\tau^c)^*$. If $\tilde{u}^2 > \tilde{u}^1$, then $(\tau^c)^* = \bar{\tau}$ and $\tilde{u} = \tilde{u}^2$, and if $\tilde{u}^1 > \tilde{u}^2$, then $(\tau^c)^* = (\tau^c)^1$ and $\tilde{u} = \tilde{u}^1$.

4.2.2 Dynamic Equilibrium

The following lemmas discuss some important results showing the convergence of the distribution of entrepreneurs to the stationary equilibrium and the potential equilibrium along the transitional path.

Lemma 4.12. *The sequence N_{Vt}^{cH} is monotone. In particular:*

(i) If $N_{V0}^{cH} < \phi^c M^H$, then N_{Vt}^{cH} is an increasing sequence converging to $\phi^c M^H$ in the stationary equilibrium.

(ii) If $N_{V0}^{cH} > \phi^c M^H$ then N_{Vt}^{cH} is a decreasing sequence converging to $\phi^c M^H$ in the stationary equilibrium.

Proof: Appendix A.3.

Lemma 4.12 shows that starting from an initial period with any given measure of high skilled agents M_0^H, the distribution of entrepreneurs converges to a stationary distribution and the equilibrium converges to a stationary equilibrium.

The following lemma shows that if the Elite's per period utility is maximized from being in a type 1 economic equilibrium in the initial period 0 and in the steady state, the economy will be in type 1 along the whole equilibrium path.

Lemma 4.13. *Let $\tilde{u_0}$ and $\tilde{u}$ be the Elite's utility maximizing period utility at the initial period and in a steady state period respectively. If $\tilde{u_0}^1 > \tilde{u_0}^2$ and $\tilde{u}^1 > \tilde{u}^2$, then it is that case that $\tilde{u_t}^1 > \tilde{u_t}^2$ $\forall t$.*

Proof: Appendix A.3.

Analysis: The role of the net external flows

Case 3 from the political equilibrium is interesting, since the value of external flows may determine the τ_t^c chosen by the Elite and, thus, the type of economic equilibrium in any period If the net external flows increase then the total net revenues for the Elite increase and they are less dependent on revenues from taxes to maintain their consumption. Given that $\delta'(\tau_t^c) > 0$, notice from the first order condition equations (33) and (35), given in its general form for a type i equilibrium by

$$(1 - \delta(\tau_t^c))[(T_t^c)^i]'(\tau_t^c) - \delta'(\tau_t^c)(TR_t)^i(\tau_t^c) = 0$$

that $(1 - \delta(\tau_t^c)) > 0$ and decreasing in τ_t^c, $[(T_t^c)^i]'(\tau_t^c) > 0$ and is decreasing for $\tau_t^c \in (0, \alpha)$. Therefore, at the given $(\tau_t^c)^i$, where $(\tilde{\tau}_t^c)^i = (\tau_t^c)^i$, any exogenous increase in $(TR_t)^i(\tau_t^c)$ at the existing policy will make the first order equations (33) and (35) negative and lower the values $(\tilde{\tau}_t^c)^i$ to satisfy both the first order conditions.

Therefore, if we are in case where $(\tau_t^c)^2 = \bar{\tau}$ and $(\tau_t^c)^1 < \bar{\tau}$, then any increase in external flows do not change the optimal policy $(\tau_t^c)^2$ for type 2, as any $(\tau_t^c)^2 < \bar{\tau}$ will not satisfy the constraint $(\tau_t^c)^2 \geq \bar{\tau}$ and $(\tau_t^c)^2 = \bar{\tau}$. Therefore, the Elite's utility $\tilde{u_t}^2$ under type 2 economic equilibrium only increases due to the increase in consumption from the extra external flows and there is no gain from an unchanged $\delta(\tau_t^c)^2$ which only depends on $(\tau_t^c)^2$.

However, the optimal policy for type 1 $(\tau_t^c)^1$ is an interior solution and any increase in $(TR_t)^1(\tau_t^c)$ will decrease $(\tau_t^c)^1$ to satisfy first order condition given by equation (33) and still be an interior solution. The effect on Elite's utility under type 1 economic equilibrium $\tilde{u_t}^1$ is two fold. First, the Elite's consumption increases from the increase in the external flows. Second, there is an increase in the Elite's probability to remain in power as $\delta(\tau_t^c)$ decreases. Thus, depending on the functional form for $\delta(\tau_t^c)$ and the exogenous parameters of the model, for a high enough level of external debt and foreign aid it may be the case that, if $(\tau_t^c)^2 = \bar{\tau}$ and $(\tau_t^c)^1 < \bar{\tau}$, then $\tilde{u_t}^2 < \tilde{u_t}^1$.

Intuitively, if external flows are high enough this decreases the Elite's dependence on tax revenues. Thus, the elite is able to lower the tax rate on the connected to increase their ability to remain in power. If they can lower it enough to get $\tau_t^c < \bar{\tau}$, this results in a type 1 economic equilibrium with misallocation and a measure of low skilled entrepreneurs entering and producing in the economy. However, when the external flows decrease the Elite's dependence on the tax revenues increases. As the tax rate for the non-connected is already at the maximum level, they can only raise tax revenues by either having larger fraction of non-connected entrepreneurs or by increasing the tax rate for the connected. Eventually, when the flows are low enough, both of these measures collectively lead to a type 2 economic equilibrium with no misallocation. Therefore, higher net external flows is more likely to perpetuate an economic equilibrium with misallocation, as they lower the $(\tau_t^c)^1$ leading to a greater increase in Elite's utility under type 1 economic equilibrium compared to type 2 economic equilibrium with no misallocation.

Also notice that at this point for case 3 it is not feasible to derive any additional properties of the potential equilibrium sequence of policies $(p^t)^*$ analytically, without having special values of the parameters and a specific functional form for $\delta(\tau_t^c)$.

5 Calibration

In this section I calibrate the parameters of the model developed in Section 4 to the case of Pakistan, assuming that the baseline calibrated economy for Pakistan is in a misallocation equilibrium along the whole equilibrium path.

Pakistan's economic landscape is heavily influenced by its dependence on external debt and a system of political patronage. Despite receiving continued support from foreign aid and external debt, Pakistan has consistently struggled with low GDP growth rates (World Bank, 2020). This is exacerbated by a chronically low tax-to-GDP ratio, indicating weak government revenue generation (Ahmed, 2019; Heritage, 2020). The country's reliance on external debt has deepened over the years, with numerous IMF bailouts required to prevent economic crises. However, these bailouts have often failed to address fundamental structural issues, such as low tax compliance and ineffective resource allocation, perpetuating Pakistan's reliance on external assistance (Cevik, 2018; FBR, 2018; Mackenzie, 2019). Moreover, Pakistan's political environment exacerbates fiscal deficits, with government expenditure often skewed towards protecting the interests of the political elite. Corruption is rampant, with politicians using their positions to extract rents and distribute them among their patronage networks, further hindering economic growth (ISAS, 2020; Jenkins & Kukutschka, 2018; Khan, 2020; UNDP, 2020). This nexus of external debt dependence and political patronage emphasizes the challenges faced by Pakistan's economy and makes it a good candidate to calibrate this model to, and to study the impact of the external flows on the system of political patronage and misallocation in the economy.

Table 2 reports the calibrated values for the parameters of the model. The discount factor β is set to a standard value of 0.99. The labour share of output $\alpha = 0.42$ is set to match Pakistan's labour share of income in 2018 as measured by the United Nations Development Program UNDP (2020). This implies that $\tau_t^n = 0.42$ for all t, which is the upper bound of the tax rates in the model.[6] I normalize low skilled ability to $A^L = 1$ and the high skilled ability A^H is set to 2.61, which is calibrated to match the skill premium ratio calculated as the ratio between the wages of most highly skilled and most low skilled categories of workers, obtained from UNDP (2020) report. As skill premium is a ratio, . The measure of connected agents ϕ^c is set to 0.3 so that the average percentage of total politically connected firms is 63%, which is equivalent to approximately the average number of political connected listed Pakistani firms for the period of 2013-2019 as reported in Table 6.[7] The probability of staying high skilled σ^H and transitioning from low to high skilled σ^L are set to match one minus the estimated ratio of the mobility of labour from high to low skilled and the mobility of labour from low to high skill workers in Pakistan respectively (Muhammad & Jamil, 2017).

No reliable data exists on estimating the costs of bribes paid or bureaucratic procedures faced by firms in Pakistan. There are several studies and papers documenting high barriers constituting of bureaucratic procedures, access to credit, lack of access to electricity, corrup-

[6]Pakistan's average corporate tax rate is 33% with 43% being the highest and 29% being the lowest in the last two decades.

[7]The average percentage of firms that are connected in a steady state equilibrium is calculated by taking the average of the politically connected firms in type 1 and type 2 steady state as a percentage of the total firms in equilibrium. This corresponds to 84% of total firms being politically connected in the steady state of baseline simulation and is consistent with the total percentage of registered firms in Pakistan in 2018 who either did not file income taxes or reported their income to be below 1700 U.S Dollars per annum.

Table 2: Calibrated Parameters

Name of the Variable (notation)	Value	Source
Discount factor (β)	0.99	Standard value in literature
Labor share of output (α)	0.42	Pakistan's labor share of income 2018[1]
High skilled ability (A^H)	2.61	Pakistan's skill premium ratio 2018[1]
Low skilled ability(A^L)	1	Normalized to 1
Probability of a high skilled to high skilled (σ^H)	0.74	Pakistan's 1-downward mobility 2012-13[2]
Probability of a low skilled to high skilled (σ^L)	0.18	Pakistan's Upward mobility 2012-13[2]
Measure of connected agents (ϕ^c)	0.30	Average measure of connected firms PSX in 2013-2019[3]
Fraction of firms with exit shock (θ)	0.7	$\phi^c + (1-\theta)\sigma^H(\frac{1}{\bar{L}} - \phi^c\frac{1}{\bar{L}}) < \frac{1}{\bar{L}}$
Calibrated Variables	**Value**	**Target Statistics**
Net external debt inflow (D)	0.06	2.9% of the GDP[4]
Foreign aid flow (F)	0.04	2% of the GDP[5]
World interest rate (r^*)	0.02	Pakistan's average interest rate on new external debt commitments[6]
Total Patronage transfer ($P^c\phi^c$)	0.18	9% of the GDP[2]
Firm size (labor employed) ($\bar{L}$)	2.8	% of firms filed income tax with middle to high income 2017-2018[7]
Fixed cost of entry (B^n)	0.76	10% of the lifetime output of a existing non-connected[8] high productivity firm
δ parameters		
(a)	25	CPI as a proxy for the slope[9]
(b)	0.3	Factor share of labour[1]

[1]UNDP (2020), [2]Muhammad & Jamil (2017), [3]Imran (2025),[4]World Bank, IDS (2020),[5]OECD, CRS (2020), [6]Pakistan Economic Survey (2020),[7]Federal Board of Revenue, Pakistan (FBR) (2017-18), [8]Afraz et al (2014), [9]Corruption Perception Index, Transparency international (2020).

tion and political instability for firms. In a study on the barriers to growth and entry for small and medium enterprises in Pakistan, Afraz et al. (2014) find that lack of access to utilities such as electricity and gas and bribes paid to obtain these facilities alone can result in loss of more than 10% of the total annual sales of a firm. In addition, bribes paid to the government officials might constitute up to an additional 4.2 percent of the total sales contract of a firm. Note that in the model the entry cost is only paid once by the new firms when they enter and these firms will not pay the entry barriers again if they do not exit entrepreneurship in their lifetime. Thus, I calibrate the fixed cost of entry B^n to match a conservative estimate of 10% of the annual lifetime before tax output of a non-connected firm conditional on remaining an entrepreneur in the model, which results in the calibrated per worker entry cost b^n of 0.27.

The model requires a functional form for delta which is increasing in τ_t^c. I consider the following functional form for $\delta(\tau_t^c)$:

$$\delta(\tau_t^c) = \frac{1}{1 + e^{-a(\tau_t^c - b)}} \tag{39}$$

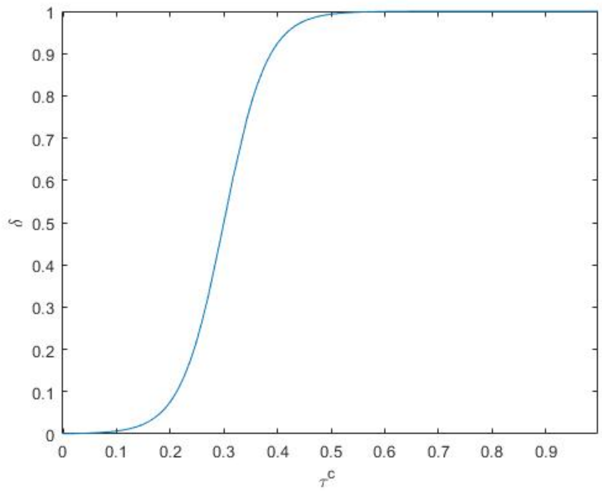


Figure 5: Functional form for $\delta(\tau_t^c)$ for $a = 25$ and $b = 0.3$

Figure 5 illustrates the functional form for $\delta(\tau_t^c)$ for the calibrated parameters. The slope of the function is controlled by the coefficient a. This slope determines how strongly the Elite's ability to stay in power depends on the privileges given to the connected in terms of lower taxes. Given the slope a, the coefficient b determines the position of $\delta(\tau_t^c)$ and influences the level of τ_t^c at which the function $\delta(\tau_t^c)$ reaches 1. The coefficients a and b collectively are chosen to match the point at which $\delta(\alpha) \approx 1$. The calibrated parameters for coefficients a and b are also consistent with Pakistan's corruption index. Pakistan's corruption level is ranked twice as that of South Korea, as measured by the corruption perception index (CPI) (Transparency International, 2018). Using this CPI ratio as the proxy for coefficient a, keeping the other parameters fixed, $a = 12.5$ is the maximum number that will result the baseline model to be calibrated to a type 2 no misallocation equilibrium. Any coefficient bigger than 12.5 would reciprocate Pakistan being in an equilibrium with misallocation. Thus, assuming $a = 12.5$ as a representation of corruption index in South Korea, $a = 25$ is consistent with being equivalent to twice of that for Pakistan.

Based on the facts related to Pakistan's environment discussed above, the rest of the exogenous parameters of the model are calibrated to a type 1 equilibrium specifications. The firm size $\bar{L}$ is set to match the percentage of registered business who filled income taxes in Pakistan with middle to high income for the year 2017-2018 as reported by the Federal Board of Revenue, Pakistan. I back out $\bar{L}$ by setting the percentage of non-connected high skilled firms to be 16% of the total firms in the baseline equilibrium. The fraction of entrepreneurs getting an exogenous exit shock θ is then set to 0.7 for which the assumption of always having some non-connected new entrepreneurs enter in equilibrium is satisfied.

There exists no statistical data regarding the firm level entry and exit rate for Pakistan, the value of 0.7 corresponds to the entry rate of new firms in a type 1 equilibrium equivalent to 12.5%, which is close to the average yearly entry rate of new firms in US for the past 10-15 years (Orazem & Winters, 2023).[8]

The external debt inflow and foreign aid D and F are calibrated to match Pakistan's average current account deficit to GDP ratio and the average foreign aid to GDP ratio for the period of (1976-2021) which are taken from World Bank (2020) and OECD (2018). A current account deficit indicates a positive net capital inflows for the country. Recently, the current account deficit in Pakistan has increased rapidly and it reached an all time high at 6.6% of GDP in 2018 and the net foreign direct inflows for Pakistan on average are declining and make less than 0.5% of Pakistan's GDP (World Bank, 2020).

The patronage amount P^c is calibrated to match the ratio of total benefit of the government expenditure as a percentage of GDP, accrued to the richest quintile in 2018, which constitutes the most politically connected income group and is obtained from UNDP (2020).

6 Baseline Model Simulation

In this section the model is solved numerically given the values of parameters calibrated above. Subsection 6.1 presents the steady state simulation results of the baseline model and is followed by Subsection 6.2 which analyses the equilibrium path for the policy and other variables of interest for the baseline model.

6.1 Steady State

Table 3 shows the baseline simulation results from Pakistan's economy being in a steady state equilibrium with misallocation. The breakdown of total output shows that the share of the total output produced by the high skilled, non-connected agents, is significantly small at 16% compared to the 84% of the output produced by all the connected agents. Out of the 84% share of the output produced by the connected, 49.5% of the output is produced by the low skilled connected entrepreneurs, which drives the misallocation in the economy. Correspondingly, the share of the low skilled connected of the total entrepreneurs is the highest at 49.5%, followed by the high skilled connected entrepreneurs at 34.5% and the lowest for the non-connected entrepreneurs at 16%. There is high discrepancy between the tax rates charged on the connected entrepreneurs 21.5% and the non-connected 42%. The

[8]Substituting in the values for the parameters in $\phi^c + (1-\phi^c)\sigma^e < \frac{1}{L} = 0.3 + (0.3)(0.74)(0.3 - 0.108) = 0.342 < 0.357$.

tax revenue to GDP ratio is 23%.[9] The Elite's probability to remain in power is quite high at 0.9 and this is due to the relatively low tax rates for the connected entrepreneurs. The Gini Coefficient for the non Elite agents in the economy is 0.5. This Gini coefficient is computed using the share of total income for each agent type in this economy. The total welfare is highest for the connected agents, all of whom are entrepreneurs in the steady state and lowest for the non-connected agents in the economy.

Table 3: Simulation Results: Baseline Model Steady State

Equilibrium Type		type 1 (misallocation)	
Aggregate Macro Variables		**Distribution of Entrepreneurs**	
Tax rate (connected firm)	21.5%	**% of Total Entrepreneurs**	
Tax rate (non-connected firm)	42%	**Connected**	
Total Output	2.07	Low skilled	49.5%
% of Total Output produced by connected	84%	High skilled	34.5%
% of Total Output produced by non-connected	16%	**Non-connected**	
Total Tax revenue	0.5	High skilled existing	3.6%
Tax/GDP	24%	High skilled new	12.4%
Gini Coefficient	0.5		
Probability to remain in power (Elite)	0.9		
Total Welfare			
Elite	45.4		
Connected	70		
Non-connected	17		

6.2 Transitional Dynamics

I consider a starting point where all agents are high skilled. As in Acemoglu (2008), this implies that in the initial period of the economy there will be some positive selection of only high skilled agents entering into entrepreneurship.

Figure 6 shows the computed value of M_t^H along the path, starting from $M_0^H = 1$, for the calibrated model, until it converges to its stationary value of $M^H = 0.41$. Notice that since the point $M_0^H > M^H$, M_t^H is a decreasing sequence. Note that in Figure 7, the green and black line represent the transitional dynamics of the corresponding variables assuming that the equilibrium is always of type 1 and type 2 respectively along the equilibrium path. The red dotted line represents the equilibrium path of the variables corresponding to the policy sequence $p^* = \{(\tau_t^c)^*\}_{t=1}^{\infty}$ that maximizes the period and lifetime welfare of the Elite as given by 29.

[9]This cannot be compared to the data where it is much lower for Pakistan due to majority of the agriculture sector being tax exempt and is beyond the scope of the single sector model in this paper. What matters is the ratio or potential gain in the tax revenues which is discussed in section 7.

Given M_t^H panel (a) in Figure 7 show the optimal policy sequence p^*. It can be seen that along the path the optimal policy at any point t is $(\tau_t^c)^* = (\tau_t^c)^1$, so that the period equilibrium is always of type 1 and $(\tau_t^c)^*$ is decreasing over time. Consequently, the Elite's ability to stay in power $\delta(\tau_t^c)$ increases overtime.

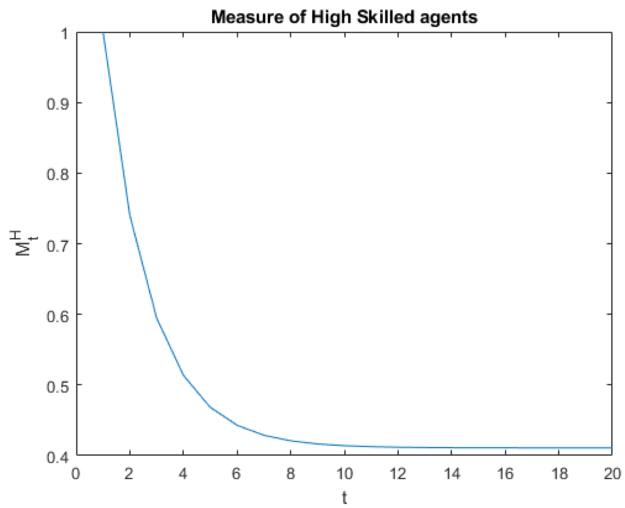
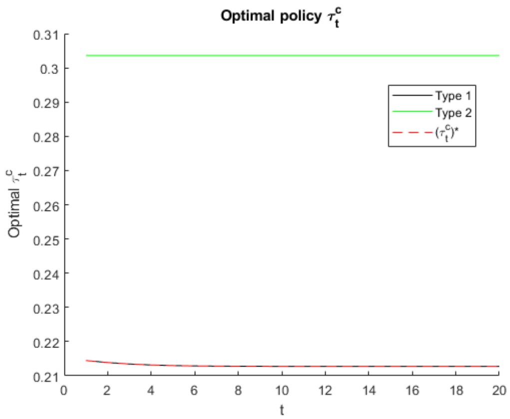


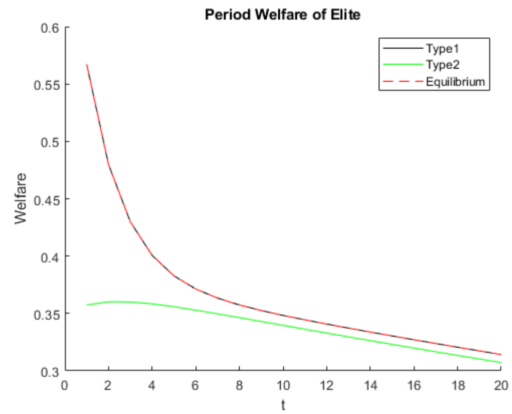
Figure 6: Measure of High skilled agents

Correspondingly, panel (b) in Figure 7 shows the period welfare of the Elite to be $(\tilde{u}_t)^* = (\tilde{u}_t)^1$ for all periods. Therefore, the lifetime welfare of the elite is maximized when the economy is always in a type 1 equilibrium along the equilibrium path.

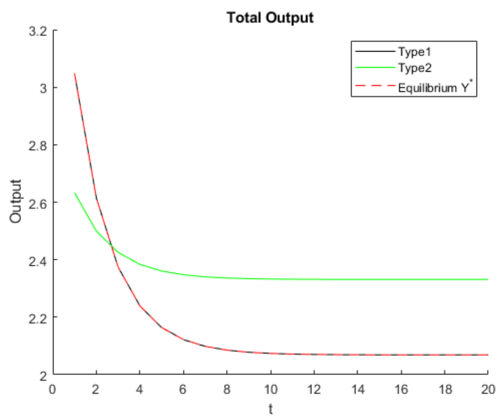
Panel (c) in Figure 7 shows that the total output $(Y_t)^*$ is decreasing over time. This is due to the decrease in the measure of high skilled agents overtime and the preferential access to the market for the connected low skilled entrepreneurs compared to the non-connected high skilled entrepreneurs, owing to a $(\tau_t^c)^*$ below $\bar{\tau}$. Thus, in panel (c) when the measure of high skilled agents falls below 0.7 the equilibrium output falls lower than it would have been under a type 2 equilibrium, but the Elite had no incentives to choose that outcome. Panel (d) in Figure 7 shows that the fraction of existing non-connected entrepreneurs in equilibrium $(N_{Vt}^{ncHe})^*$ starting from $N_{V0}^{ncHe} = 0$ remains stationary at approximately 0.013.



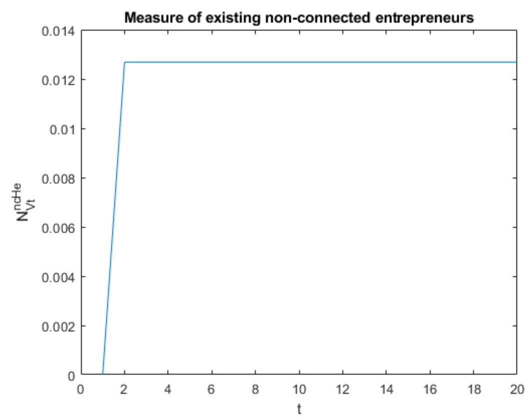
(a) Optimal policy τ_t^c



(b) Period Welfare Elite



(c) Total Output



(d) Non-connected Existing Entrepreneurs

Figure 7: Transitional Dynamics Baseline Simulation

The next section discusses the impact on the key results for the baseline economy from carrying out counterfactual analysis by reducing the level of the external debt.

7 Counterfactual Exercises

In this section, I perform counterfactual analysis to determine the effect of a reduction of external flows in the economy. Subsection 7.1 presents the steady state results from the counterfactual exercise and is followed by Subsection 7.2 which shows the impact of the transitional dynamics, including the policy and key variables of interest in the model.

7.1 Steady State: Reduction in External Flows

Table 4 shows the results of the baseline model by reducing the level of external debt by 30% and keeping the foreign aid F and other parameters fixed at the values of the baseline calibration. The simulation results show that under the lower external debt, the economy is in type 2 equilibrium with no misallocation. It is also note worthy to mention that in a minimum 30% decrease in the total external flows $(D + F)$ from the initial period is enough for the economy to converge to a type 2 stationary equilibrium with higher total output.

The changes in the aggregate macro variables show that the output of the economy increases and there is a significant gain of 12% compared to the benchmark model simulation. There is a substantial increase of 309% in the output produced by the non-connected high skilled agents and a corresponding decrease of only 49% in the output produced by the connected entrepreneurs.

This is also reflected in the change in the composition of entrepreneurs. There are no low skilled entrepreneurs in the economy and the total share of non-connected entrepreneurs increases to 65.5%. There is also a significant increase in the fraction of existing non-connected entrepreneurs. The existing non-connected entrepreneurs now consist of around 14.5% of the total entrepreneurs in the economy, an 11% increase from the baseline model.

The probability of being removed from power increases by 5 times, but the welfare of the Elite is maximized from being in type 2 equilibrium as there are higher revenues from taxes and entry barriers which can compensate for the loss in welfare from the decrease in the net external debt inflow. This reaffirms the presumptions that a decrease in external debt causes the Elite to be more dependent on the economy's own resources. To maintain it's revenues the Elite increases the tax rate on non-connected agents to $\tau_t^c = 0.30$, making it unprofitable for the low skilled connected to become or remain entrepreneurs. It is noteworthy to see that the model predicts a gain of about 1.5 times in the tax revenues to GDP ratio compared to the baseline results, which is within the maximum tax capacity of Pakistan estimated to be approximately twice the actual tax to GDP ratio for 2018 by Fenochietto & Pessino (2013).

The share of the income is now more equally distributed as suggested by the lower Gini Coefficient compared to the baseline model. This is due to the fact that now there is less favorable treatment towards the connected entrepreneurs in terms of lower taxes and there is more equal opportunity for the non-connected high skilled entrepreneurs to enter and produce. The life time welfare of the non-connected agents increase by 32.4%. This is because there are now a higher percentage of existing non-connected entrepreneurs, who do not have to pay the entry costs, thus making higher profits. In comparison there is a decrease in the welfare of the Elite of 5.2%, due to a decrease in its consumption from lower flows and an increase in the probability of losing power due to higher τ_t^c. Similarly, there is also a decrease in the welfare of the connected of about 23% due to connected entrepreneurs paying higher taxes and the low skilled connected becoming workers.

Table 4: Counterfactual Results: Reduced External Flows Steady State

$D = 0.03$			
Equilibrium Type		**type 2** (no misallocation)	
Aggregate Macro variables		**Distribution of Entrepreneurs**	
Tax rate on connected	30.3%	**% of Total Entrepreneurs**	
Δ Gain in Output	12%	**Connected**	
Δ Loss in the Output by connected	31%	Low skilled	0%
Δ Gain in the Output by non-connected	309%	High skilled	34.5%
Δ Gain in Tax revenues	74%	**Non-connected**	
Gain in Tax to GDP ratio	1.5 times	High skilled existing	14.5%
Gini Coefficient	0.38	High skilled new	51%
Loss in the probability to remain in power (Elite)	5 times		
Change in Total Welfare			
ΔElite	-5.2%		
Δ Connected	-23%		
Δ Non-connected	32.4%		
Minimum reduction in the external flows to be in a steady state of Type 2	30%		

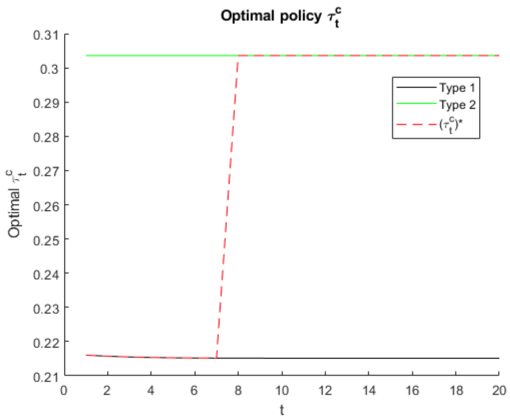
7.2 Transitional dynamics

Figure 8 shows the change in the equilibrium path of the key variables, starting with $M_0^H = 1$, when D is reduced by 30% from the initial period.

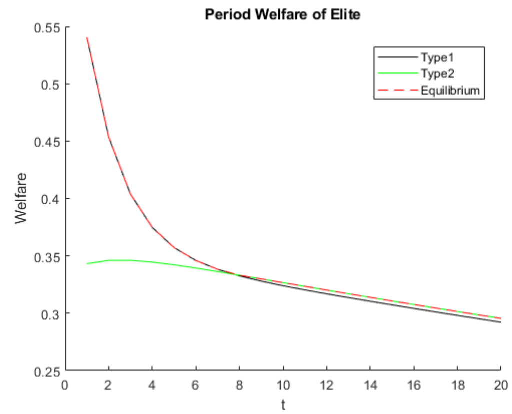
Notice from Figure 8 panel (a) and (b), for the first seven periods the Elite maximizes its period welfare by selecting the optimal policy such that $(\tau_t^c)^* = (\tau_t^c)^1$ so that the economy remains in the type 1 equilibrium. From period $t = 8$ onwards, the Elite's period welfare is maximized such that $(\tau_t^c)^* = (\tau_t^c)^2$ and the economy switches to equilibrium of type 2 and remains in type 2 equilibrium, converging to a type 2 steady state.

Panel (c) in Figure 8 shows that for the first two periods the economy remain in type 1 equilibrium and does not lose the advantage of higher output due to higher proportion of connected entrepreneurs being high skilled and paying less taxes and investing higher capital compared to type 2. In aggregate given the parameter values the economy stays below its maximum possible output for about five periods before transitioning and converging to an equilibrium type 2 with higher output in the steady state.

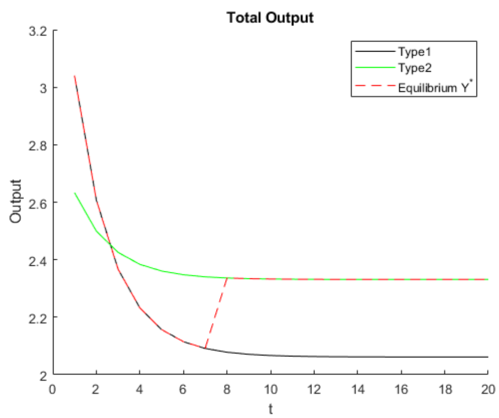
Panel (d) in Figure 8 shows that the measure of existing non-connected entrepreneurs remain low for the first five periods before sharply increasing when the economy switches to being in type 2 and steadily increase until it converges to the type 2 steady state. Notice that it takes one extra period for the existing non-connected entrepreneurs to increase after the economy transitions to type 2 equilibrium. This is because the measure of existing non-connected entrepreneurs depends on the measure of total non-connected entrepreneurs in the previous period.



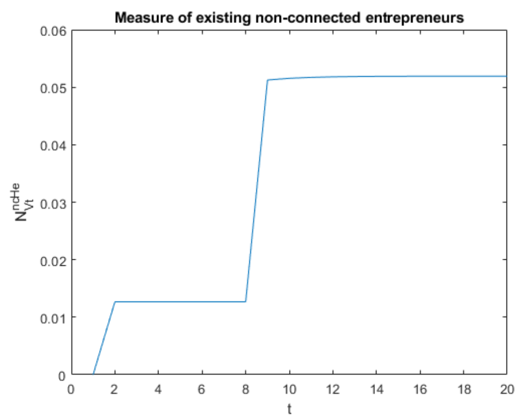
(a) Optimal policy τ_t^c



(b) Period Welfare Elite



(c) Total Output



(d) Non-connected Existing Entrepreneurs

Figure 8: Transitional Dynamics Reduced External Debt

Notice that any decrease in the external flows greater than 30% will increase the welfare of the non connected high skilled entrepreneurs by a higher percentage. This is because the lower the non-tax revenues for the Elite, the quicker will be the transition to an equilibrium with no misallocation (type 2). Correspondingly, the output and the measure of existing non connected entrepreneurs earning positive profits will also increase sooner contributing to the increase in their welfare. Appendix A.5 provides the results of simulations with D=0 corresponding to a 60% decrease in the external flows.

8 Sensitivity Analysis

Some of the calibrated parameters in the baseline model are not based on clear estimates but on certain proxies due to data restrictions and availability. In this section for robustness checks of the calibrated parameters, I perform sensitivity analysis by changing some of the key calibrated parameters to see the impact on the steady state results of the benchmark model reported in Table 2. The analysis is performed by changing one key parameter at a time, keeping the other parameters fixed. The results of the sensitivity analysis are summarized in Table 5. It can be observed that the main results of the baseline model do not change much with respect to small changes in the parameter values of $\bar{L}$, a, ϕ^c. However, the results of the model are sensitive to the changes in the values of the entry barriers b^n.

Table 5: Sensitivity Analysis

Baseline model	Type 1	τ^c =21.5%	
Parameters	**Type**	$\boldsymbol{\tau^c}$	**Output gain in Type 2**
$\boldsymbol{\bar{L}}$			
2.5	Type 1	20.4%	8.4%
2.8	Type 1	21.5%	12% (Baseline model)
3.0	Type 1	22.4%	12.5%
$\boldsymbol{b^n}$			
0.29	Type 1	20.2%	11%
0.27	Type1	21.5%	12% (Baseline model)
0.26	Type 2	29.6%	None
$\boldsymbol{\delta(\tau^c)}$			
$\boldsymbol{a}$			
a = 20	Type 1	19.3%	7.3%
a=25	Type 1	21.5%	12% (Baseline model)
a = 30	Type 1	23.8%	13.2%
$\boldsymbol{\phi^c}$			
.28	Type1	20.6%	9.6%
.30	Type 1	21.5%	12% (Baseline model)
.32	Type 1	22.5%	13.4%

I first change the value of the firm size to the maximum and the minimum values obtained from the parameter restrictions arising from assumption 1 of the model on the measure of the connected and the total high skilled agents in the economy. It can be noticed that decreasing and increasing the firm size to 2.5 and 3.0 respectively does not change the type of equilibrium and the potential output gain from switching to a type 2 equilibrium for the baseline economy. This is followed by changing the value of the entry barriers. It can be observed that increasing entry barriers do not impact the equilibrium but lowering the entry barriers below 0.27 results in the baseline economy being already in type 2 stationary equilibrium. This is an expected result of the model as we start with the premise of an economy with high entry barriers.

Changing the slope of the functional form for the Elite's ability to being removed from power by 25% below and above the calibrated value for the baseline model, does not change the equilibrium type and the potential gain in the output for the simulated economy. Lastly, changing the measure of connected entrepreneurs by 6% below and above the calibrated value, also does not significantly impact the main results of the baseline model.

The sensitivity analysis reaffirms that the simulated model results are not sensitive to small changes in the key parameter values, except the entry barriers. This reinforces the role of institutional entry barriers to entrepreneurship as one of the key factors contributing to resource misallocation in developing economies.

9 Discussion: Policy Recommendations

This paper's theoretical and quantitative analysis underscores the importance of attaching conditions to external debt and aid to address resource misallocation in developing countries with corrupt political systems reliant on patronage. The results suggest that unconditional external flows perpetuate inefficiencies, including fiscal policies that promote low-productivity entrepreneurship and resistance to institutional reforms that reduce barriers to entry for new firms.

The findings indicate that limiting the amount of unconditional aid and debt can incentivize governments to rely more on domestic resources. For example, a 30% reduction in Pakistan's external debt is projected to enhance total economic output. Additionally, external debt and foreign aid programs tied to minimum direct tax revenue targets can encourage fiscal responsibility. The transitional dynamics presented in Figure 12 in the Appendix A.5 shows that in Pakistan, increasing direct tax revenue by 66% of its current level would result in a steady-state equilibrium with no misallocation while maintaining existing aid levels.

Lastly, external debt and foreign aid programs should prioritize reforms that lower entry barriers for new firms, as even a modest 4% reduction in entry barriers per worker could transition Pakistan's economy to a steady-state equilibrium without misallocation, as demonstrated by Figure 13 in the Appendix A.5. Such targeted conditions would enhance economic growth, reduce inequality, and address persistent inefficiencies in developing economies.

10 Conclusion

This paper demonstrates how unconditional external debt and foreign aid can exacerbate resource misallocation and hinder growth in developing countries. Using Pakistan as a case study, I show both theoretically and quantitatively that high external flows, coupled with political patronage, perpetuate low growth and high inequality. Empirical evidence reveals that politically connected firms in Pakistan benefit from lower effective tax rates, and this preferential treatment intensifies with an increase in public external debt to GDP ratio.

The political economy model developed in this paper explains these findings, showing that connected entrepreneurs receive tax advantages in exchange for political support. The model highlights how resource misallocation arises when low-skilled, politically connected individuals enter into entrepreneurship. It also shows that the level of external flows can determine whether the economy operates with or without resource misallocation. A key result is that reducing unconditional external flows diminishes tax differentials between the connected and non-connected entrepreneurs and increases economic output, as demonstrated by the calibrated model.

This study highlights the need for international donor organizations to structure aid and debt programs with conditions that require recipient nations to increase direct tax revenues or reduce entry barriers for firms. These measures can eliminate resource misallocation and foster economic growth. While this paper focuses on tax-related preferential treatment, future research should explore the broader effects of other forms of favoritism on economic growth and determine the optimal level of external flows to support development in the presence of such treatments.

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A Appendix

A.1 Further details of the empirical analysis

Sample Selection Details The sample selection process starts with searching for operational Pakistani firms, publicly listed and private, with available financial reports on the S & P platform as of the financial year end of 2019. Information was available on 433 Pakistani firms as of 2019, out of which 343 were non-financial and did not belong to the public sector. Following the standard practice in empirical finance, financial firms are excluded from the sample (Fama & French, 1992).[10] Of the 343 firms, 261 listed non-financial firms had most of the key financial information available from 2012 to 2019. Some of the missing key financial variables for these 261 firms were supplemented from the individual company's annual reports from the State Bank of Pakistan (SPB), their individual websites and other online sources (State Bank of Pakistan, 2021). There was missing information for the remaining 82 firms that could not be supplemented from other sources. However, for 7 of these 82 firms, there was missing information on key variables only for some years, which were imputed using the average growth rate of the variables from the prior years. I conduct the primary empirical analysis using a sample of a balanced panel of 268 firms.[11] I also report the results from a sample of 261 firms, excluding the firms with imputed values for robustness exercises in the next section.

Distribution of firms Table 6 presents the final distribution of the sample firms based on connection. It shows that given the definition of politically connectedness for the sample with 268 firms 63.4% of the firms are identified as politically connected for the year 2013-2017 and 63.8% of the firms are identified as politically connected for the years 2018-2019. The distribution of connected firms is quite similar excluding the firms with extrapolated values. This is suggestive of the fact that there a significant proportion of firms in Pakistan have directors associated with politics and it is consistent over the years. Further more, the table shows that 18 additional firms become politically connected after the general election of 2018 and 17 firms lost their political connections as none of their key board of directors took part in the 2018 general elections for both the samples.

[10] This is mainly due to differences in the leverage structure of the financial firms, which can impact the sensitivity to interest rate and subsequently their taxable income and valuation.

[11] Note that some survival bias may exist due to the exclusion of some of the firms for which the data was not available for the entire period and the inability to distinguish between firms that exited the market or simply delisted to become private equity firms with no publicly available information or annual reports.

Table 6: Sample Distribution of firms based on Connections

Total number of firms with available data	261	
Years (Period)	**2013-2017**	**2018-2019**
No. Connected firms per year	168	169
No. non-connected firms per year	93	92
% of connected firms per year	**64.4%**	**64.8%**
Total observations per year	261	261
Total observations for 2013-2019	1827	

Total number of firms with available and imputed data	268	
Years (Period)	**2013-2017**	**2018-2019**
No. Connected firms per year	170	171
No. non-connected firms per year	98	97
% of connected firms per year	**63.4%**	**63.8%**
Total observations per year	268	268
Total observations for 2013-2019	1876	
Firms with changed connectivity status for 2018-2019		
No of firms with POLCON 0 to 1	18	
No of firms with POLCON 1 to 0	17	

Table 7: Descriptive statistics 1

Variables	**mean**		**sd**		**min**		**max**	
No of firms	261	268	261	268	261	268	261	268
ETR	0.207	0.204	0.301	0.300	0	0	1	1
POLCON	0.645	0.635	0.479	0.481	0	0	1	1
EDPGDP	0.252	0.252	0.045	0.045	0.215	0.215	0.347	0.347
EDGDP	0.295	0.295	0.064	0.064	0.238	0.238	0.436	0.436
Control Variables								
SIZE	1.126	1.136	0.827	0.826	-1.948	-1.948	3.366	3.366
COLLATERAL	0.556	0.543	0.227	0.238	-0.041	-0.041	1	1
ROA	0.044	0.044	0.134	0.133	-2.792	-2.792	0.421	0.421
GOVGDP	0.113	0.113	0.003	0.003	0.108	0.108	0.117	0.117
LIR	0.102	0.102	0.016	0.016	0.082	0.082	0.127	0.127
FX	0.009	0.009	0.001	0.001	0.007	0.007	0.01	0.01

Note: ETR = (Tax expenses- Deferred tax expenses)/(Earnings before interest rate and tax); POLCON=1 if the firm has a board of director who is politically connected; 0 otherwise; EDPGDP = (public and publicly guaranteed external long term debt stock+ short term external debt stock in US dollars)/ (Nominal GDP in US dollars) ; EDGDP = (Total external debt stock in US dollars)/(Nominal GDP in US dollars); SIZE= Log of Total Assets; COLLATERAL= (Total Assets- Total current Assets)/(Total Assets); ROA= (Earnings before interest and tax)/(Total Assets); GOVGDP= (Total government expenditure in US dollars)/(Nominal GDP in US dollars); LIR= Annual average SBP lending interest rate; FX= Average annual foreign exchange rate pf Pakistani Rupee in terms of US dollars.

Table 7 presents the summary statistics for the final dependent and the explanatory variables that are used in the analysis for the two samples containing 261 firms and the 268 firms. The table shows that the mean ETR for the final cleaned and recoded variables in the sample is around 20% for all the firms. The mean EDGDP Total External Debt to GDP ratio is 29.5% for Pakistan for the period of 2013-2019, with the mean EDPGDP public external debt to GDP ratio being 25.2%. The public external debt to GDP ratio for Pakistan which comprises of more than 80% of the county's total external debt fluctuates between 21.5% and 34.7% during the sample period.

A.2 Robustness Checks

This section presents a number of additional checks all of which supports the empirical results presented and methodology adopted in this paper.

Political Strength: Being Connected vs Winning

Table 8 presents the results from the analysis to see whether politically connected firms who have candidates which win the elections pay even lower effective tax rates. A political connected firm who won an election is represented by the time-variant dummy variable POLWIN, which takes value one if the firm has at least one politically connected person who won an election in its board of director and zero otherwise. Column (1) and (2) represents the results for the sample of 268 firms using the variable POLWIN for the following specifications:

$$ETR_{it} = \alpha_0 + \beta_1 \cdot POLWIN_{it} + \lambda \cdot X_{it} + \psi_i + \pi_t + \epsilon_{it} \tag{40}$$

$$\begin{aligned} ETR_{it} = \alpha_0 + \beta_1 \cdot POLCON_{it} + \beta_2 \cdot POLWIN_{it} + \beta_3 \cdot POLCON_{it} * POLWIN_{it} \\ + \lambda \cdot X_{it} + \psi_i + \pi_t + \epsilon_{it} \end{aligned} \tag{41}$$

It can be seen from column (1) that for specification 40 the coefficient on the variable $POLWIN$ is negative but not significant. Similarly, the results from specification 41 presented in column (2) shows that collectively the coefficients on $POLWIN$ and the interaction term $POLCON * POLWIN$ are significant and offset their respective effects, overall indicating a very negligible effect on the effective tax rates of a politically connected firm with a board of director who won an election compared to the politically connected firm without one. These results support the definition of politically connected firms used for the primary analysis in this paper suggesting that, in Pakistan, it is sufficient for a vital member of a firm to be running as a candidate in a particular election to receive preferential treatment in terms of paying a lower effective tax rate and that winning an election does not strengthen this effect. However, the insignificant result for the $POLWIN$ variable in specification 40

Table 8: Political Strength: Connectedness vs Winning

	(1) ETR	(2) ETR
POLCON		-0.0834**
		(0.0339)
POLWIN	-0.0233	-0.581**
	(0.0325)	(0.212)
SIZE	-0.00845	0.00033
	(0.0392)	(0.0391)
COLLATERAL	-0.148**	-0.146**
	(0.0639)	(0.0638)
ROA	-0.290***	-0.293***
	(0.0652)	(0.0650)
POLCON*POLWIN		0.576**
		(0.214)
Firm fixed effects	Yes	Yes
Time fixed effects	Yes	Yes
No of Observations	1876	1876

Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Variable definitions: ETR = (Tax expenses- Deferred tax expenses)/(Earnings before interest rate and tax); POLCON=1 if the firm has a board of director who is politically connected; 0 otherwise; POLWIN=1 if the firm has a board of director who is politically connected and won the election; 0 otherwise; SIZE= Log of Total Assets; COLLATERAL= (Total Assets-Total Current Assets)/(Total Assets); ROA= (Earnings before interest and tax)/(Total Assets).

may also arise because the number of politically connected firms that won an election is relatively low, representing a small sample size.

Exclusion of Imputed Variables and the Effect of Total External Debt

Table 9 presents the additional analysis for the sample with 261 firms in columns (1) and (5) for specification 1 and 2. Columns (3) and (4) present the results for the two samples using the total external debt to GDP ratio (EDGDP) for the following specification:

$$ETR_{it} = \alpha_0 + \beta_1 \cdot POLCON_{it} + \beta_2 \cdot EDGDP_t + \beta_3 \cdot POLCON_{it} * EDGDP_t \\ + \lambda_1 \cdot X_{it} + \lambda_2 \cdot X_t + \psi_i + \epsilon_{it} \quad (42)$$

It shows that results are not sensitive to the exclusion of the additional 7 firms and that the preferential treatment effect is less strong when using total external debt to GDP ratio. It can be noticed that the coefficients for $POLCON$, $POLCON * EDPGDP$ and $POLCON * EDGDP$ are not significantly different between the two samples. Also note that

Table 9: Total External Debt vs Public External Debt

	(1)	(2)	(3)	(4)	(5)	(6)
	ETR	ETR	ETR	ETR	ETR	ETR
No of firms	261	268	261	268	261	268
POLCON	-0.0832**	-0.0837**	0.135*	0.127*	0.175**	0.168**
	(0.0341)	(0.0339)	(0.0749)	(0.0734)	(0.0876)	(0.0857)
EDGDP			-0.442			
			(0.839)			
POLCON*EDGDP			-0.681***	-0.656***		
			(0.208)	(0.203)		
EDPGDP					0.0359	0.0858
					(0.761)	(0.747)
POLCON*EDPGDP					-0.953***	-0.926***
					(0.298)	(0.290)
SIZE	-0.0216	-0.00458	-0.0269	-0.0259	-0.0283	-0.0273
	(0.0416)	(0.0390)	(0.0414)	(0.0405)	(0.0414)	(0.0406)
COLLATERAL	-0.166**	-0.150**	-0.163**	-0.159**	-0.163**	-0.159**
	(0.0646)	(0.0656)	(0.0637)	(0.0638)	(0.0643)	(0.0639)
ROA	-0.283***	-0.293***	-0.276***	-0.272***	-0.274***	-0.270***
	(0.0663)	(0.0651)	(0.0661)	(0.0656)	(0.0661)	(0.0657)
GOVGDP			-0.141	-0.781	-4.807	-4.885
			(8.646)	(8.503)	(6.713)	(6.600)
LIR			-0.318	-0.458	-1.164	-1.200
			(1.168)	(1.148)	(0.821)	(0.807)
FX			0.00292	0.00271	0.00212	0.00199
			(0.00234)	(0.00230)	(0.00208)	(0.00205)
Firm fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Time fixed effects	Yes	Yes	No	No	No	No
No of Observations	1827	1876	1827	1876	1827	1876

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Variable definitions: ETR = (Tax expenses- Deferred tax expenses)/(Earnings before interest rate and tax); POLCON=1 if the firm has a board of director who is politically connected; 0 otherwise; EDPGDP = (public and publicly guaranteed external long term debt stock+ short term external debt stock in US dollars)/ (Nominal GDP in US dollars) ; EDGDP = (Total external debt stock in US dollars)/(Nominal GDP in US dollars); SIZE= Log of Total Assets; COLLATERAL= (Total Assets-Total Current Assets)/(Total Assets); ROA= (Earnings before interest and tax)/(Total Assets); GOVGDP= (Total government expenditure in US dollars)/(Nominal GDP in US dollars); LIR= Annual average SBP lending interest rate; FX= Average annual foreign exchange rate of Pakistani Rupee in terms of US dollars.

for the specification (2) using public and publicly guaranteed external debt, the coefficient β_3 for $POLCON * EDPGDP$ is significantly higher than the coefficient for the specification using the total external debt stock $POLCON * EDGDP$.

Figures 9-10 shows that the difference between the predictive effective tax rates for the politically connected (polcon=1) and non-connected firms (polcon=0) is larger, when there is an increase in the public external debt to GDP ratio, compared to the increase in total external debt to GDP ratio. This supports the argument that an increase in the external debt provided to the government leads to a greater increase in the preferential treatment for the politically connected firms in terms of lower effective tax rates compared to the total external debt which includes the private long term external debt.

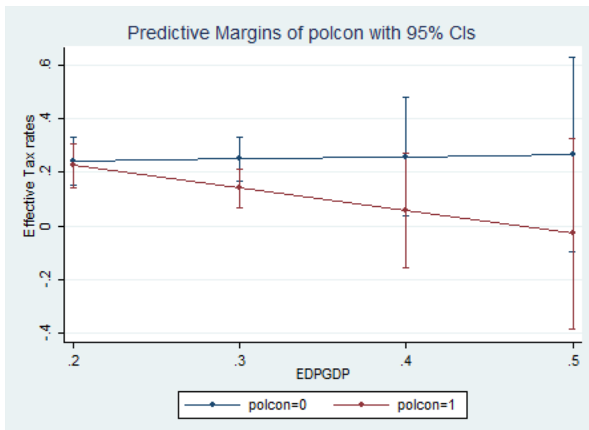


Figure 9: Predicted Effective Tax Rates by Public External Debt to GDP ratio

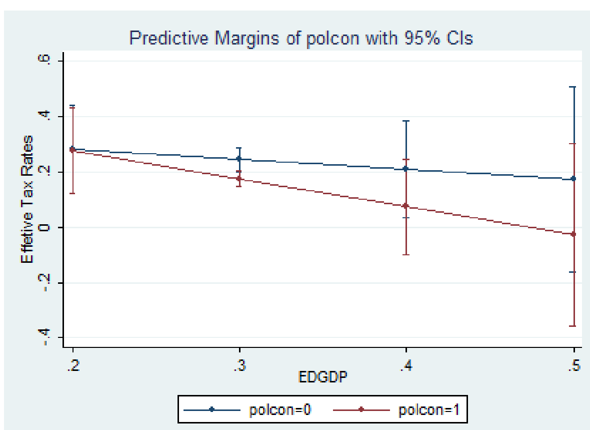


Figure 10: Predicted Effective Tax Rates by External Debt to GDP ratio

A.3 Proofs

***Lemma* 4.1.** Given the evolution of skills described in expression 3, we can write the measure of high skilled agents M^H_{t+1} at period $t+1$ recursively as:

$$M^H_{t+1} = \sigma^H M^H_t + \sigma^L(1 - M^H_t)$$

A stationary point satisfies

$$M^H_{t+1} = M^H_t = M^H \quad \forall t$$

Then

$$M^H = \sigma^H M^H + \sigma^L(1 - M^H)$$
$$M^H = \frac{\sigma^L}{(1 - \sigma^H + \sigma^L)} = \bar{\sigma}$$

□

***Lemma* 4.2.** Given M^H_0 let us compute:

$$M^H_1 = \sigma^H M^H_0 + \sigma^L(1 - M^H_0) = M^H_0(\sigma^H - \sigma^L) + \sigma^L$$
$$M^H_2 = M^H_0(\sigma^H - \sigma^L)^2 + \sigma^L(1 + (\sigma^H - \sigma^L))$$
$$M^H_3 = M^H_0(\sigma^H - \sigma^L)^3 + \sigma^L(1 + (\sigma^H - \sigma^L) + (\sigma^H - \sigma^L)^2)$$
$$M^H_4 = M^H_0(\sigma^H - \sigma^L)^4 + \sigma^L(1 + (\sigma^H - \sigma^L) + (\sigma^H - \sigma^L)^2 + (\sigma^H - \sigma^L)^3)$$

Following this pattern, in general, at period t:

$$M^H_t = M^H_0(\sigma^H - \sigma^L)^t + \sigma^L \sum_{s=0}^{t-1} (\sigma^H - \sigma^L)^s$$

Taking the limits in this expression as $t \to \infty$ and given that $(\sigma^H - \sigma^L) \in (0, 1)$, the first term converges to zero and the second term converges to $\frac{\sigma^L}{1-\sigma^H+\sigma^L}$. Therefore M_t converges to $\frac{\sigma^L}{1-\sigma^H+\sigma^L} = \bar{\sigma}$. □

***Lemma* 4.3.** To prove (i) I will first show that $M^H_t < M^H_{t+1}$, when $M^H_0 < \bar{\sigma}$ and then show that $M^H_t < M^H_{t+1} < \bar{\sigma}$.
Given that $\bar{\sigma} = \frac{\sigma^L}{(1-\sigma^H+\sigma^L)}$. Let us first show that for $t = 1$, $M^H_0 < M^H_1$ and $M^H_1 < \bar{\sigma}$, when

$M_0^H < \bar{\sigma}$. Proof by contradiction. Lets suppose $M_0^H \geq M_1^H$. Then:

$$M_0^H \geq M_1^H = \sigma^L + (\sigma^H - \sigma^L)M_0^H$$
$$\text{Rearranging and gathering terms}$$
$$M_0^H(1 - \sigma^H + \sigma^L) \geq \sigma^L$$
$$M_0^H \geq \frac{\sigma^L}{(1 - \sigma^H + \sigma^L)} = \bar{\sigma}$$

Which contradicts the assumption that $M_0^H < \bar{\sigma}$.

Similarly, to show that $M_1^H < \bar{\sigma}$. Lets suppose $M_1^H \geq \bar{\sigma}$. Then:

$$M_1^H = \sigma^L + (\sigma^H - \sigma^L)M_0^H \geq \frac{\sigma^L}{(1 - \sigma^H + \sigma^L)}$$
$$M_0^H \geq \left(\frac{\sigma^L}{(1 - \sigma^H + \sigma^L)} - \sigma^L\right)\frac{1}{\sigma^H - \sigma^L}$$
$$M_0^H \geq \left(\frac{\sigma^H - \sigma^L}{(1 - \sigma^H + \sigma^L)}\right)\frac{1}{\sigma^H - \sigma^L} = \frac{\sigma^L}{(1 - \sigma^H + \sigma^L)} = \bar{\sigma}$$

Which again contradicts the assumption that $M_0^H < \bar{\sigma}$. Thus the statement is true for $t = 1$.

Now assume that $M_t^H < \bar{\sigma}$. Then using a similar argument as before. Lets suppose $M_t^H \geq M_{t+1}^H$. Then:

$$M_t^H \geq M_{t+1}^H = \sigma^L + (\sigma^H - \sigma^L)M_t^H$$
$$\text{Rearranging and gathering terms}$$
$$M_t^H(1 - \sigma^H + \sigma^L) \geq \sigma^L$$
$$M_t^H \geq \frac{\sigma^L}{(1 - \sigma^H + \sigma^L)} = \bar{\sigma}$$

Which contradicts the assumption that $M_t^H < \bar{\sigma}$.

Similarly, to show that $M_{t+1}^H < \bar{\sigma}$, first assume that $M_t^H < \bar{\sigma}$. Lets suppose $M_{t+1}^H \geq \bar{\sigma}$. Then:

$$M_t^H = \sigma^L + (\sigma^H - \sigma^L)M_t^H \geq \frac{\sigma^L}{(1 - \sigma^H + \sigma^L)}$$
$$M_t^H \geq \left(\frac{\sigma^L}{(1 - \sigma^H + \sigma^L)} - \sigma^L\right)\frac{1}{\sigma^H - \sigma^L}$$
$$M_t^H \geq \left(\frac{\sigma^H - \sigma^L}{(1 - \sigma^H + \sigma^L)}\right)\frac{1}{\sigma^H - \sigma^L} = \frac{\sigma^L}{(1 - \sigma^H + \sigma^L)} = \bar{\sigma}$$

Which again contradicts the assumption that $M_t^H < \bar{\sigma}$. Thus the statement is true for all t.

Therefore, by induction if $M_0^H < \bar{\sigma}$ then M_t^H is a strictly increasing sequence, such that $M_t^H < M_{t+1}^H < \bar{\sigma}$.

To prove (ii) a similar argument to the previous case shows that if $M_0^H > \bar{\sigma}$ then M_t^H is a strictly decreasing sequence, such that $\bar{\sigma} < M_{t+1}^H < M_t^H$.

To prove (iii), I show that $M_0^H = M_1^H$ and $M_t^H = M_{t+1}^H = \bar{\sigma}$

$$M_1^H = \sigma^L + (\sigma^H - \sigma^L)M_0^H$$

We know that $M_0^H = \bar{\sigma}$

Substituting for M_0^H in the expression for M_1^H and rearranging we get

$$M_1^H = \frac{\sigma^L}{(1 - \sigma^H + \sigma^L)} = M_0^H = \bar{\sigma} = M^H$$

Let us assume that $M_t^H = \bar{\sigma}$, then:

$$M_{t+1}^H = \sigma^L + (\sigma^H - \sigma^L)M_t^H$$

We know that $M_t^H = \bar{\sigma}$

Substituting for M_t^H in the expression for M_{t+1}^H and rearranging we get

$$M_{t+1}^H = \frac{\sigma^L}{(1 - \sigma^H + \sigma^L)} = M_t^H = \bar{\sigma} = M^H$$

Therefore, by induction if $M_0^H = \bar{\sigma}$ then M_t^H is a constant sequence, such that $M_t^H = M_{t+1} = \bar{\sigma}$. □

Lemma 4.4. Based on the above wage thresholds (19)-(21) given that $A^H > A^L$ and $b^n > 0$, it is the case that wage thresholds are monotonically increasing in skill level z and decreasing in b^n, so that w_t^{cH} is always the highest and w_t^{nLr} is always the lowest and $w_t^{nze} > w_t^{nzr}$. □

Lemma 4.6. Using the measures of agents and the assumption 1 it can be shown that for: **Cases 1 and 2**: All connected agents become entrepreneurs. Given assumption 1 (b) $\phi^c < \frac{1}{L}$ it is the case that the total measure of connected are less than the total number of firms in equilibrium. Given assumption 1 (c) $M_t^H > \frac{1}{L}$ the total number of connected high skilled agents plus the non-connected high skilled agents are greater than the total number of firms. Finally, given assumption 1(d) $N_0^e = 0$ and 1(f) $\phi^c + \sigma^H(1-\theta)(\frac{1}{L} - \frac{1}{L}\phi^c) < \frac{1}{L}$ total number of non-connected existing entrepreneurs plus the total connected at any time t is less than the total number of equilibrium firms, this would make the equilibrium wage equal to w_t^{nHr} where, $NG_t^{nHr} = 0$ so that few but not all non-connected high skilled workers become entrepreneurs. Otherwise, if the wages are below w_t^{nHr} there will be excess demand of labor as all the non-connected high skilled agents would strictly prefer to become entrepreneurs and if they are above w_t^{nHr} there will an excess supply of labor as all non-connected high skilled non existing entrepreneurs would want to remain workers .

Cases 3: Given assumption 1 (c) $M_t^H > \frac{1}{L}$ the total number of connected high skilled plus the non-connected high skilled agents are greater than the total number of firms. Finally, given assumption 1(d) $N_0^e = 0$ and 1(f) $\phi^c + \sigma^H(1-\theta)(\frac{1}{L} - \frac{1}{L}\phi^c) < \frac{1}{L}$ and $(1-\sigma^H) > \sigma^L$ the total number of existing entrepreneurs plus the high skilled connected entrepreneurs at any time t is less than the total number of firms, this would make the equilibrium wage equal to w_t^{nHr} where, $NG_t^{nHr} = 0$ so that few but not all non-connected high skilled workers become entrepreneurs. Otherwise, if the wages are below w_t^{nHr} there will be excess demand of labour as all the non-connected high skilled agents would strictly prefer to become entrepreneurs and if they are above w_t^{nHr} there will an excess supply of labour as all non-connected high skilled non existing entrepreneurs would want to remain workers. □

Lemma 4.7. Using that $w_t^* = w_t^{nHr}$ it is the case that $\forall t$, non-connected high skilled individuals who were not entrepreneurs in the previous period are indifferent between becoming a worker or an entrepreneur. Then in equilibrium the net value gain of becoming an entrepreneur for a non-connected individual with a non-existing firm ownership is NG_t^{nHr} =0, or

$$V_{t+1}^{nH} - b^n\bar{L} = W_{t+1}^{nH}$$

which implies

$$V_{t+1}^{nH} > W_{t+1}^{nH}$$

so that non existing non-connected high skilled agents are indifferent between being an entrepreneur and all the high skilled non-connected existing entrepreneurs at time t always remain entrepreneurs. Assumption 3 then implies that low skilled non-connected entrepreneurs will always choose to remain workers so that,

$$V_{t+1}^{nL} < W_{t+1}^{nL}$$

Using these two :

$$CV_{t+1}^{nH} = \sigma^H V_{t+1}^{nH} + \sigma^{1-H} W_{t+1}^{nL}$$

and

$$CW_{t+1}^{nH} = \sigma^H (V_{t+1}^{nH} - b^n\bar{L}) + \sigma^{1-H} W_{t+1}^{nL}$$

Thus, $CV^{nH}_{t+1} - CW^{nH}_{t+1} = \sigma^H b^n$. □

Proposition 1. The differentiation of the conditions (1)-(2) follows directly from comparing the wage thresholds w^{nHr}_t and w^{nLc}_t given in lemma 4.5 and the derivations in lemma 4.7. Notice that measures of connected high skilled entrepreneurs depend on the transitional dynamics given in expression (3). In particular, (d) and (e) depend on N^{nH}_{Vt-1}, which in turn depends on whether $\tau^c_{t-1} \geq \bar{\tau}$ or $\tau^c_{t-1} < \bar{\tau}$. □

Proposition 2. The differentiation of the conditions **(1s)-(2s)** follows directly from comparing the wage thresholds w^{nHr}_t and w^{nLc}_t given in lemma 4.5 and the derivations in lemma 4.7. Notice that measures of connected high skilled entrepreneurs depend on the transitional dynamics given in expression (3), specifically the measure of high skilled agents. Starting from an initial level of high skilled agents in the economy M^H_0, lemmas 4.1-4.2 show that M^H_t converges to a stationary point M^H. In particular, (d) and (e) depend on N^{nH}_V and N^{cL}_V, which in turn depends on whether $\tau^c \geq \bar{\tau}$ or $\tau^c < \bar{\tau}$. □

Lemma 4.8. It follows from proposition 1 :

1. If $\tau^c_t < \bar{\tau}$ the measure of total connected entrepreneurs is $N^{cH}_{Vt} + N^{cL}_{Vt} = \phi^c \ \forall t$. Therefore, the total measure of non-connected high skilled entrepreneurs is $N^{nH}_{Vt} = \frac{1}{\bar{L}} - \phi^c$ and is a constant.

2. If $\tau^c_t \geq \bar{\tau}$ the measure of total connected entrepreneurs is $N^{cH}_{Vt} = \phi^c M^H_t$ and $N^{cL}_{Vt} = 0$ $\forall t$. Therefore, the measure of non-connected high skilled entrepreneurs is $N^{nH}_{Vt} = \frac{1}{\bar{L}} - \phi^c M^H_t$.

Therefore, the tax revenues in period t if $\tau^c_t < \bar{\tau}$ can be written as:

1. $(T^c_t)^1 = \frac{1}{1-\alpha}\tau^c_t(1-\tau^c_t)^{\frac{1-\alpha}{\alpha}}\bar{L}[\phi^c M^H_t A^H + \phi^c(1-M^H_t)A^L]$ from the connected entrepreneurs.

2. $(T^n_t)^1 = \frac{1}{1-\alpha}\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}\bar{L}(\frac{1}{\bar{L}} - \phi^c)A^H$ from the non-connected entrepreneurs.

The tax revenues in period t if $\tau^c_t \geq \bar{\tau}$ can be written as:

1. $(T^c_t)^2 = \frac{1}{1-\alpha}\tau^c_t(1-\tau^c_t)^{\frac{1-\alpha}{\alpha}}\bar{L}[\phi^c M^H_t A^H]$ from the connected entrepreneurs.

2. $(T^n_t)^2 = \frac{1}{1-\alpha}\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}\bar{L}(\frac{1}{\bar{L}} - \phi^c M^H_t)A^H$ from the non-connected entrepreneurs.

□

Lemma 4.9. Optimum is at the corner in both problems. □

Lemma 4.10. The proof follows from proposition 1. Given assumption 2 it is the case that $\tau_t^c \leq \alpha$. It can be noticed that the total measure of high skilled non-connected entrepreneurs N_{Vt}^{nH}, who pay higher tax ($\tau_t^n = \alpha$), is higher under type 2 equilibrium, compared to the type 1 equilibrium. Correspondingly, the measure of low skilled connected entrepreneurs N_{Vt}^{cL} is zero under type 2 equilibrium and the measure of high skilled connected N_{Vt}^{cH} is the same under both type 1 and type 2 equilibrium. This implies that for any given τ_t^c the total tax revenues for the Elite will be higher under a type 2 equilibrium and the total revenues will also be higher, that is $(TR_t)^2(\tau_t^c) > (TR_t)^1(\tau_t^c)$. □

Lemma 4.11. By contradiction. Assume that $(\tau_t^c)^1 < \bar{\tau}$. Comparing the FOC for the two problems, it follows from proposition 1 and lemma 4.10 that $(TR_t)^2(\tau_t^c)^1 > (TR_t)^1(\tau_t^c)^1$ and $((T_t^c)^2)'(\tau_t^c)^1 < ((T_t^c)^1)'(\tau^c)^1$. In general, $(1 - \delta(\tau_t^c)) > 0$ and decreasing in τ_t^c and notice from the first order conditions given by (33) and (35) that, $(T_t^c)'(\tau_t^c) > 0$ and is decreasing for $\tau_t^c \in (0, \alpha)$ and $\delta'(\tau_t^c) > 0$. Therefore, at the given $(\tau_t^c)^1$ the FOC for type 2 economic equilibrium given by (35) is negative, as the measure of connected entrepreneurs are lower and of the non-connected are higher, paying higher taxes. Thus, the unconstrained interior solution that satisfies (35), will be lower than $(\tau_t^c)^1$, that is $(\tau_t^c)^2 < (\tau_t^c)^1 < \bar{\tau}$. Therefore, $(\tau_t^c)^2 < \bar{\tau}$, which contradicts the assumption. □

Proposition 3.

1. If $(\tau_t^c)^1 = (\tau_t^c)^2 = \bar{\tau}$, $\delta(\bar{\tau})^1 = \delta(\bar{\tau})^2$. It follows from proposition 1 and lemma 4.10 that $(TR_2)^2(\bar{\tau}) > (TR_2)^1(\bar{\tau})$. Thus, $\tilde{u_2}^2 > \tilde{u_t}^1$. The economic equilibrium at period t is of type 2 and the optimal equilibrium policy $(\tau_t^c)^* = \bar{\tau}$.

2. The proof follows from proposition 1 and lemmas 4.10 and 4.11. Lemma 4.10 and case 1 shows that if $(TR_t)^2(\bar{\tau}) > (TR_t)^1(\bar{\tau})$ then $\tilde{u_t}^2 > \tilde{u_t}^1$. Lemma 4.11, shows that if $(\tau_t^c)^2 > \bar{\tau}$ then $(\tau_t^c)^1 = \bar{\tau}$, which means that $(\tau_t^c)^2$ is an interior solution and $(\tau_t^c)^1$ is a corner solution. Also, we know that Elite's utility under type 2 equilibrium $\tilde{u_t}^2$ is higher when $(\tau_t^c)^2 > \bar{\tau}$ compared to the case when $(\tau_t^c)^2 = \bar{\tau}$. Thus, if $\tilde{u_t}^2 > \tilde{u_t}^1$ when $(\tau_t^c)^1 = (\tau_t^c)^2 = \bar{\tau}$, then it is always the case that $\tilde{u_t}^2 > \tilde{u_t}^1$ and $\tilde{u}_t = \tilde{u_t}^2$, when $(\tau_t^c)^1 = \bar{\tau}$ is a corner solution and $(\tau_t^c)^2 > \bar{\tau}$ is an interior solution.

□

Proposition 4. Arguments similar to the proof for proposition 3. □

Lemma 4.12. It follows directly from assumption 1 and lemmas 4.1-4.2. □

Lemma 4.13. Given that the initial measure of entrepreneurs is zero and given lemma 4.8 the Elite's per period tax revenue is only a function of M_t^H which is exogenously determined and τ_t^c. If $\tilde{u_0}^1 > \tilde{u_0}^2$ and $\tilde{u}^1 > \tilde{u}^2$, given proposition 3 this implies $(\tau_0^c)^2 = \bar{\tau}$, $(\tau_0^c)^1 < \bar{\tau}$

and $(\tau_0^c)^* = (\tau_0^c)^1$ in period 0, and $(\tau^c)^2 = \bar{\tau}$, $(\tau^c)^1 < \bar{\tau}$ and $(\tau^c)^* = (\tau^c)^1$ in a steady state period. The monotonicity and convergence of the sequence M_t^H follows from lemmas 4.1-4.2 which implies that in this case $\{(\tau_t^c)^1\}_{n=t}^{\infty}$ is either a decreasing or an increasing sequence and $\{(\tau_t^c)^2\}_{n=t}^{\infty} = (\bar{\tau}^c)\}_{n=t}^{\infty}$ a constant sequence, as this holds true for period 0 and in the steady state. Thus, if the Elite maximizes their per period initial and steady state utility from being in a type 1 equilibrium, then for the entire equilibrium path $\tilde{u_t}^1 > \tilde{u_t}^2$. □

A.4 Economic Equilibrium Additional Details

***Derivation of Assumption 3** that eliminates cases 4 and 5.* Given $\tau_t^n = \alpha$ and lemma 4.6 the equilibrium wage $w_t^* = w_t^{nHr}$, for cases 1, 2 and 3, which satisfies,

$$NG_t^{nHr} = 0$$

and

$$V_t^{nH} - W_t^{nH} - b^n \bar{L} = 0 \quad \forall t$$

Similarly, if cases 3 and 4 were allowed, given the assumption 1 and that every period θ fraction of existing entrepreneurs die. For, cases $w_t^{nHr} \geq w_t^{nLe}$ or $w_t^{nHr} < w_t^{nLe}$, the equilibrium wage will always be $w_t^* = w_t^{nHr}$. This is because some non-connected new high skilled entrepreneurs will always be required to enter in the equilibrium, as the total number of existing entrepreneurs will be less than the total firms $\frac{1}{L}$ required for market clearing in equilibrium. I have shown that $w_t^* = w_t^{nHr}$ for cases 1, 2, 3, 4 and 5 $\forall t$, given $\tau_t^n = \alpha$, I can then write,

$$V_t^{nj} = V_{t+1}^{nj} = V^{nj}$$

Therefore if $w_t^{nHr} > w_t^{nLe}$ then,

$$V^{nL} - W^{nL} < V^{nH} - W^{nH} - b^n \bar{L} = 0$$

where,

$$V^{nL} - W^{nL} = \frac{\alpha}{1-\alpha}(1-\alpha)^{1/\alpha}(A^L - A^H) + b^n - \beta(1-\theta)\sigma^H b^n < 0$$

Thus, it follows that if,

$$\frac{\frac{\alpha}{1-\alpha}(1-\alpha)^{1/\alpha}(A^H - A^L)}{1-\beta(1-\theta)\sigma^H} > b^n$$

then,

$$w_t^{nHr} > w_t^{nLe}.$$

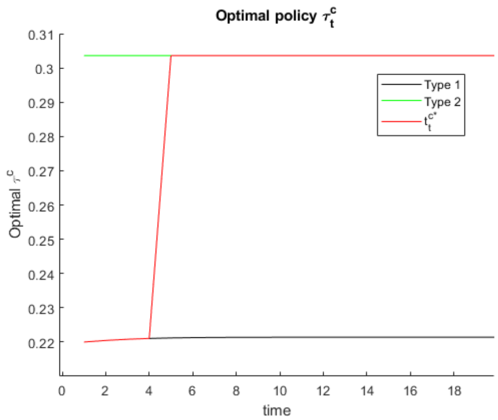
□

A.5 Additional Simulations

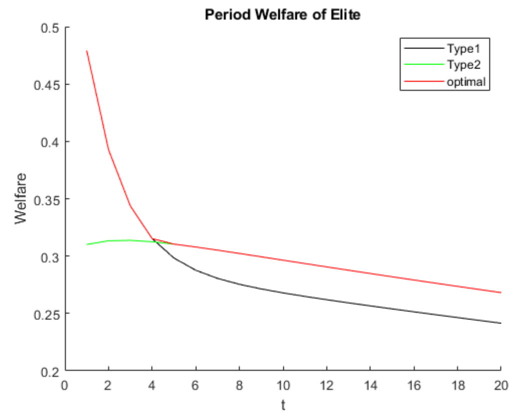
This section presents the simulation results from a counterfactual exercise of eliminating the external debt and setting $D = 0$. Table 10 shows the change in the total welfare for the Elite, the non-connected and connected agents. Notice that the gain in the welfare for the non-connected is higher and loss in the welfare for the Elite and the connected is also higher under a larger decrease in the level of external flows. Figure 11 shows the simulations from the transitional dynamics of eliminating the external debt. It can be noticed that the lower the external flows the sooner the economy transitions to a type 2 (no misallocation) equilibrium.

Table 10: Welfare Analysis Reduced External flows $D = 0$

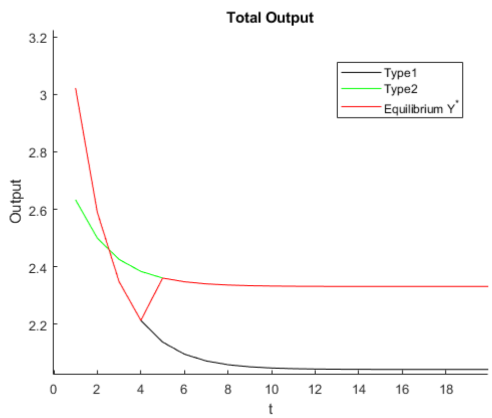
Equilibrium Type $D = 0$	**type 2** (no misallocation)
Change in Total Welfare	
ΔElite	-11%
Δ Connected	-21.6%
Δ Non-connected	47%



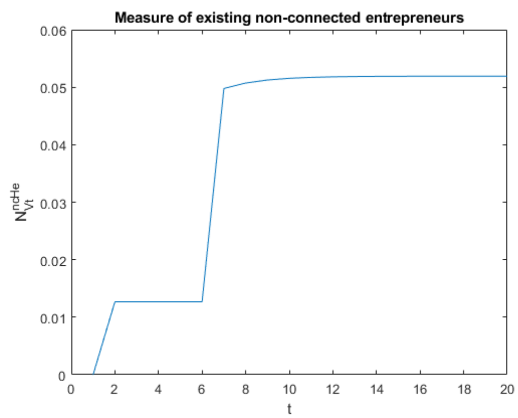
(a) Optimal policy τ_t^c



(b) Period Welfare Elite

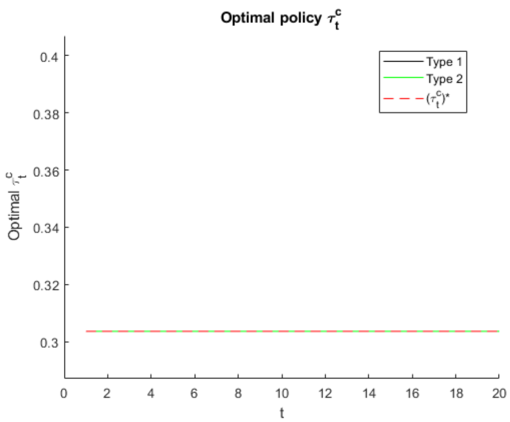


(c) Total Output

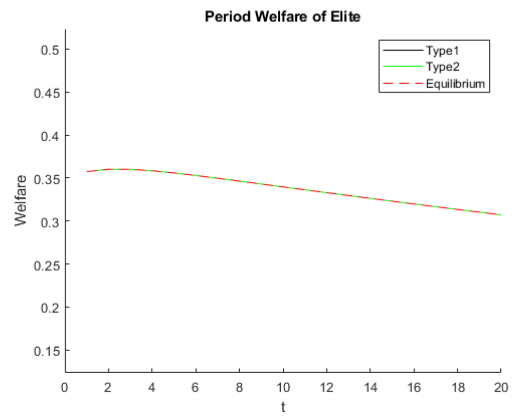


(d) Non-connected Existing Entrepreneurs

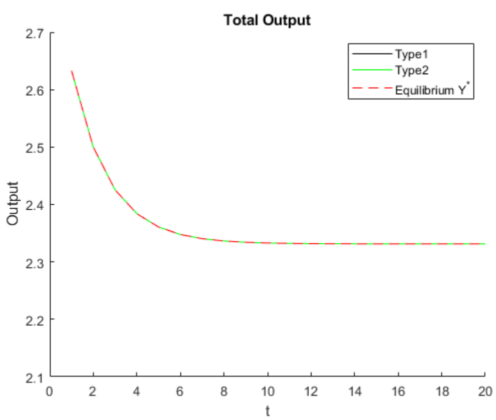
Figure 11: Transitional Dynamics Baseline Simulation $D = 0$



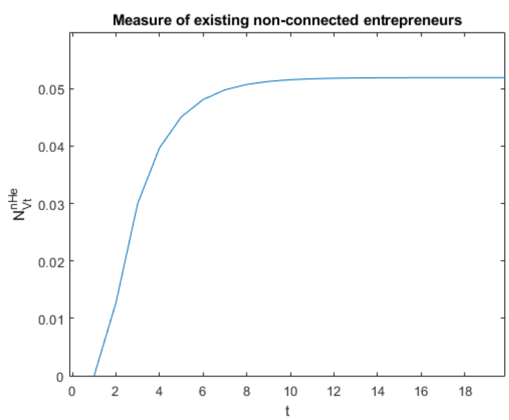
(a) Optimal policy τ_t^c



(b) Period Welfare Elite

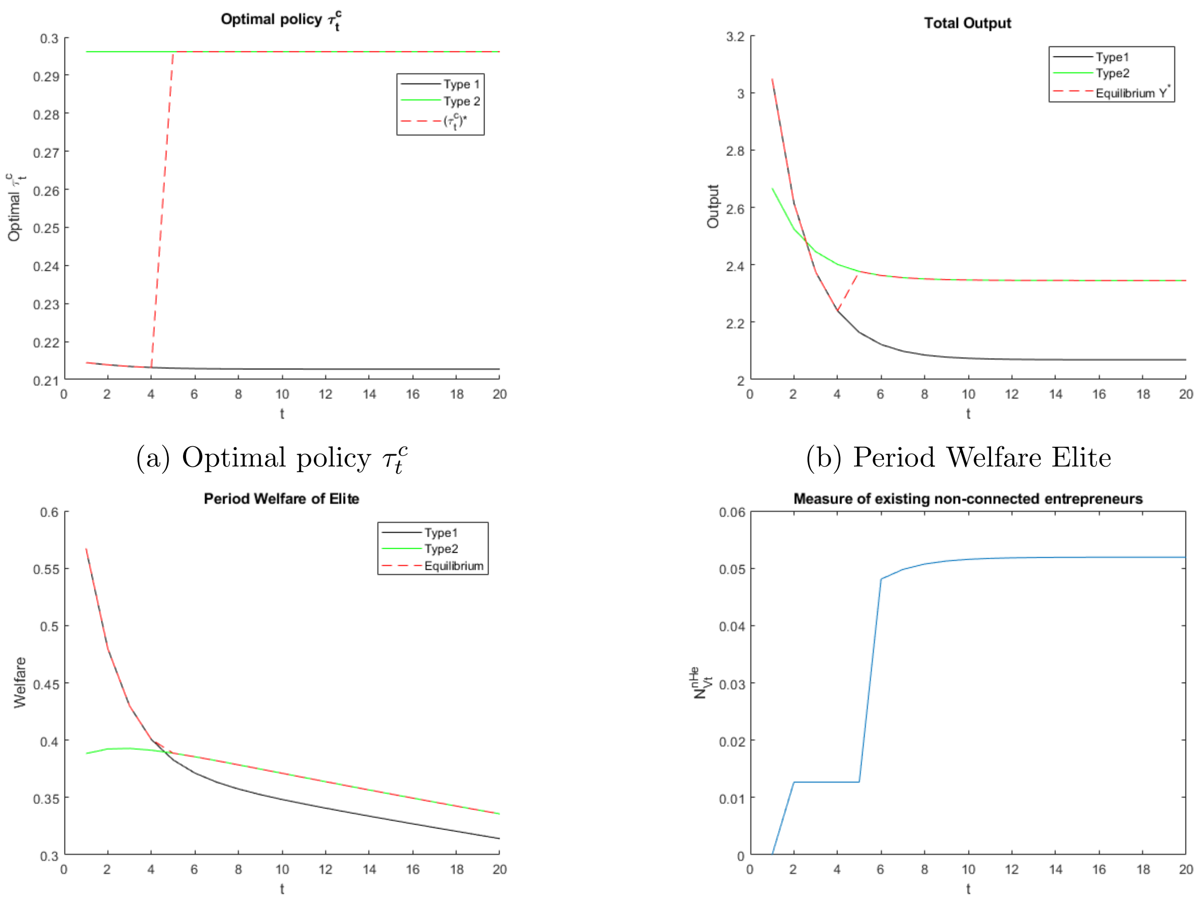


(c) Total Output



(d) Non-connected Existing Entrepreneurs

Figure 12: Transitional Dynamics: Tax Revenue Constraint



(a) Optimal policy τ_t^c

(b) Period Welfare Elite

(c) Total Output

(d) Non-connected Existing Entrepreneurs

Figure 13: Transitional Dynamics: Lower Entry Barrier B^n